Algebraical Exercises

AND

EXAMINATION PAPERS

BY

C. A. BARNES, M.A.

PRICE, 30 CENTS.

TORONTO
THE COPP, CLARK COMPANY, LIMITED

THE LIBRARY

UNIVERSITY OF
WESTERN ONTARIO

THE J. D. BARNETT
TEXT-BOOK COLLECTION

University of Western Ontario
LIBRARY
LONDON - CANADA

Class LT1001
512.9 .B26 1897?

PLEASE

DO NOT REMOVE THIS
CARD FROM BOOK POCKET

LIBRARIES

THE UNIVERSITY OF WESTERN ONTARIO

LONDON CANADA

Digitized by the Internet Archive
in 2009 with funding from
Ontario Council of University Libraries

http://www.archive.org/details/algebraicalexerc00barn

ALGEBRAICAL EXERCISES

— AND —

EXAMINATION PAPERS

FOR PUBLIC SCHOOL LEAVING AND PRIMARY EXAMINATIONS

BY

C. A. BARNES, M.A.

Inspector of Schools, Lambton.

TORONTO:
THE COPP, CLARK COMPANY, LIMITED.

Entered according to Act of the Parliament of Canada, in the year one thousand eight hundred and ninety-seven, by The Copp, Clark Company, Limited, Toronto, Ontario, in the Office of the Minister of Agriculture.

1306

PREFACE.

This book of Exercises in Algebra has been prepared to supply a want felt by many teachers who are teaching elementary Algebra, and for private students. The ordinary text has not a sufficient number of Examples to enable students to become thoroughly familiar with the Principles of Elementary Algebra, and expert in the best methods of solution. The aim of this book is to supply that deficiency, and the *Hints* given in the Answers on the methods of solution, it is believed, will prove helpful to many private students, and enable them to lay a good foundation for more advanced work.

The book is particularly intended for students who are preparing for either P. S. Leaving or Primary Examinations.

THE AUTHOR.

EXERCISES IN ALGEBRA.

EXERCISE I.

ADDITION.

1. $7a+5b+3c$; $9a+6b+5c$; $14a-7b-6c$; $3b-9c$; $4a+5b$.

2. $8ax-7by+3y^2$; $ax^2+2by-7y^2$; $9ax+4y^2-2$; $3by-2y^2+7$; $4ax-y^2+6ax'$.

3. $\frac{1}{2}a-\frac{1}{3}b+\frac{1}{5}c$; $\frac{1}{4}a-\frac{1}{5}b-\frac{1}{3}c$; $\frac{1}{3}a+\frac{1}{4}b+\frac{1}{2}c$; $b-a+c$.

4. $5a-2b+3c-4d$; $3b-4c+5d-2a$; $5c-6d+3a-4b$; $7d-4a+5b-4c$.

5. $p+q-r$; $-(q+r-s)$; $r+p-s$; $-(p-s-r-q)$.

6. $2ap-3xy+4mn$; $5mn+3xz+7xy$; $3mn-5c^3+2ap-4ap-4xy-12mn$.

7. $4a-10b+13c-2d$; $a+6b-14c+5d$; $3a-17b+6c+14d$; $a+14b-c-17d$.

8. $4(a^2+b^2)+37(ab+ac)+7(a^2+b^2)+9(ab+ac)+9(abc+a^2b^2c^2)-11(a^2+b^2)+15(ab+ac)+61(abc+a^2b^2c^2)-61(ab+ac)$.

9. $3x+y+z$; $-7x-4y+11z$; $4z-x-y$; $11x+y-z$.

10. $\frac{a}{b}-2\frac{p}{q}$; $3\frac{a}{b}-7\frac{p}{q}$; $5\frac{a}{b}+9\frac{p}{q}-14\frac{a}{b}+7\frac{a}{b}$.

EXERCISE II.

SUBTRACTION.

1. From $17a-13x+27$ take $15a-11x+9$.

2. From $4a^2-7xy-7y^3+5z^2+13m-11$,
take $3a^2+7xy-9y^3+z^2+12m-11$.

3. From $4a-7c+5xy^2-7\sqrt{a-b^2}$,
take $-12a+7c-9xy^2+6\sqrt{a-b^2}$.

4. From $p^2+3q^2+11r^2+15pq-7y-1$,
take $20q^2+11r^2+17pq-7y-100$.

5. What expression must be taken from $2a^3-16a^2b+6ab^2-2b^3$ to leave $a^3-17a^2b-3ab^2$?

6. From $\frac{5}{2}x^2-\frac{1}{3}xy-\frac{3}{2}y^2$,
take $\frac{3}{2}x^2+xy-\frac{3}{2}y^2$.

7. From the sum of $14a+9b-13c$ and $6a+5b-11c$ take their difference.

8. Subtract $3x^3+4x^2y-7xy^2+20y^3$ from $4x^3-2x^2y+4xy^2+14y^3$.

9. From $\frac{3}{4}a^2-\frac{5}{6}a+\frac{4}{5}$,
take $\frac{1}{2}-\frac{4}{3}a-\frac{a^2}{4}$.

10. Subtract the sum of $\frac{13}{6}a-\frac{1}{3}c$ and $\frac{1}{2}c+\frac{1}{6}b-\frac{8}{3}a$ from the sum of $\frac{1}{2}c-a$ and $\frac{2}{12}b-\frac{1}{3}c$.

EXERCISE III.

ADDITION AND SUBTRACTION.

1. Add $11a-b+c$; $2a+2b-3c$; $-3a+4b+14c$ together and take $14a+5b-15c$ from the sum.

2. Add $12-x+x^2$; $3x-14-13x^2$; $15+12x^2+7x-x^2$ and from their sum take $11-11x-3x^2$.

3. Add $7x-11+\sqrt[3]{xyz}$; $16-y+18\sqrt[3]{xyz}$; $15y-6x-20\sqrt[3]{xyz}$ and $4z-14y+9x-20$ together and take $5x-11y-5z+2\sqrt[3]{xyz}-15$ from the sum.

4. What quantity added to a^2-b^2 will give x^2+y^2 ?

5. If $x=2a+3b$; $y=3a-2b$; $z=b-4a$, show that $3x+8y+4z=14a-3b$.

6. Add $2a^2-b^2+\frac{c^2}{6}$; $3a^2+b^2-\frac{c^2}{3}$; $b^2-a^2+\frac{c^2}{6}$ together and from the sum subtract $4a^2+b^2-c^2$.

7. Add $\frac{a^2}{4}+\frac{b^2}{3}-\frac{c^2}{2}$; $\frac{b^2}{2}+\frac{c^2}{4}-\frac{a^2}{2}$; $\frac{c^2}{2}+\frac{a^2}{4}-\frac{b^2}{3}$ and from the sum take $\frac{b^2}{2}+\frac{c^2}{4}$.

8. From $\frac{a^2+b^2}{2}$ take $\frac{a^2-b^2}{2}$.

9. Add $13a+15b-c$; $2a-14b+c$; $\frac{b}{2}-10a+\frac{c}{3}$ and from the sum subtract $\frac{a}{2}-\frac{b}{2}+\frac{c}{3}$.

10. Add $a(a+b-c)$; $b(b+c-a)$; $c(a+c-b)$.

EXERCISE IV.

Remove brackets and collect like terms.

1. $a-b-c-(a+3b-c)+2a-(b-2c+a-b-c)$.
2. $2a-x+c-(3a-x)-(4x-5a)+3-2x-(a-x)+3x-2$.
3. $a-\{2b-(3c+2b-a)\}$.
4. $2a-b-\{-(c-d)-(-2a+b+d)\}$.
5. $3a-(a-b-c)-2\{a+c-2(b-c)\}$.
6. $3a-[a+b-\{a+b+c-(a+b+c+d)\}]$.
7. $\{a-(b-c)\}-\{b-(c-a)\}-\{c-(a-b)\}-(a+b+c)$.
8. $2a-[3b-\{-2c-(2a-\overline{2c+3b})\}]$.
9. $11p-\{3q+(2r-\overline{3s-7p})-8q\}-[4r+\{3s-(2p+q)\}]$.
10. $4[a+3\{b-c-4(d-e+f)\}]-[3a+4(3b-3c-12d+12e-12f)]$.

EXERCISE V.

MULTIPLICATION.

A.

1. Multiply $x^2-2xy+y^2+x-y$ by $x-y$.
2. " $x^2+y^2+z^2+xy-xz+yz$ by $x-y+z$.
3. " $1+2x+3x^2+6x^3$ by $1-2x$.
4. " $16x^4-8x^3y+4x^2y^2-2xy^3+y^4$ by $2x+y$.
5. " $a^2-2ab+b^2+c^2$ by $a^2+2ab+b^2-c^2$.
6. " $x^3+5x^2-16x-1$ by $x^3-5x^2-16x+1$.
7. " $1+x+x^2+x^3+x^4+x^5+x^6$ by $1-x$.
8. " $x^3-3x^2a+3xa^2-a^3$ by $x^3+3x^2a+3xa^2+a^3$.
9. " x^m+y^n by x^m+y^n.
10. " $a^3-a^2b+ab^2-b^3$ by $a+b$.

B.

1. Multiply $(a+c)^2+2(a+c)(b+d)+(b+d)^2$ by $a+b+c+d$.
2. " a^4-2a^2+4 by a^4+2a^2-4.
3. " $9x^2+4y^2+z^2-2yz-3xz-6xy$ by $3x+2y+z$.
4. " $1-y-z+y^2-yz+z^2$ by $1+y+z$.

5. Multiply $ax+a^2x^2+a^3x^3$ by $1-ax$.

6. 〃 $a^4+2a^2b^2+b^4$ by $a^4-2a^2b^2+b^4$.

7. 〃 $x^2+y^2+z^2-2yz$ by $x^2-y^2-z^2+2yz$.

8. 〃 $x^8+x^6+x^4+x^2+1$ by x^2-1.

9. 〃 $a+mx-nx^2$ by $a-mx+nx^2$.

10. Find the continued product of $x+\frac{a}{2}$; $x-\frac{a}{2}$; $x^2+\frac{a^2}{4}$; $x^4+\frac{a^4}{16}$.

C.

1. Multiply $x^8-x^4y^4+y^8$ by x^4+y^4.

2. Find the coefficient of x^3 in the product of x^2-ax+p by x^2+bx+q.

3. Multiply $x^2-(a+b)x+ab$ by $x^2+(a+b)x+ab$ and examine what the product becomes if in it either a or b is substituted for x.

4. Write down without actual multiplication the product of $x^3-a^2y+y^3$ and $x^3-a^2y-y^3$.

5. Multiply $x+a-b$ by $x-a+b$ and show that the product is zero if the difference between a and b is equal to x.

6. Find the product of $(x-a)(x-b)(x-c)$.

7. From number 6 find the coefficient of x in the product of $(x+8)(x+3)(x-2)$.

8. From number 6 write down the product of $(x-1)(x-2)(x-3)$.

9. Multiply $(x-3)(x-5)(x-7)$ using the principles of number six.

10. Multiply $(x+a)(x+b)(x+c)(x+d)$ and deduce from the product the coefficient of x in the product of $(x+2)(x+6)(x+10)(x+14)$.

EXERCISE VI.

DIVISION.

A.

1. Divide $20a^3-15a^2+25a$ by $5a$.

2. 〃 $a^4-2a^2x^2+x^4$ by $a^2+2ax+x^2$.

3. 〃 $x^6-x^4+x^3-x^2+2x-1$ by x^2+x-1.

4. 〃 $x^2+ax+bx+ab$ by $x+a$.

5. 〃 $x^3-(a+2)x^2+(2a+b)x-2b$ by $x-2$.

6. 〃 $a^2-b^2-c^2+2bc$ by $a-b+c$.

7. Divide $24x^2-65xy+21y^2$ by $3x-7y$.

8. " $x^4+10x^3+35x^2+50x+24$ by $x+4$.

9. " $x^4+3x^2y^2+2y^4$ by x^2+2y^2.

10. " $x^3+3ax^2+3a^2x+a^3+b^3$ by $x+a+b$.

B.

1. Divide $(a+b)^2+3(a+b)c+2c^2$ by $a+b+c$.

2. Show that $x^5+3x^4-x^3-x^2-2$ is divisible by $x+1$ and $x-1$, and write down the quotient.

3. Write down the quotient of $x^3+8y^3-27z^3+18xyz$ by $x+2y-3z$, and test the result by putting $x=5$; $y=-4$; $z=3$ in divisor, dividend and quotient.

4. Divide $(a+b)^2-(a+b)(c+d)-6(c+d)^2$ by $a+b+2(c+d)$.

5. Divide the product of a^2+ax+x^2 and a^3+x^3 by $a^4+a^2x^2+x^4$.

6. Divide $x^3+(a+b+c)x^2+(ab+ac+bc)x+abc$ by $x+a$.

7. If the dividend be $4a^2b^2+2(3a^4-2b^4)-ab(5a^2-11b^2)$ and the quotient $2a(a+b)+(a^2-b^2)$, what is the divisor ?

8. By what must a^4+a^2+1 be multiplied to make it a^8+a^4+1 ?

9. The product of two algebraical expressions is $x^6+x^5y+x^4y^2-x^3y^3+y^6$ and one of them is x^2+xy+y^2 ; what is the other ?

10. What value of a will make $6x^4-2x^3+2ax^2+2x+a$ exactly divisible by x^2-x+1 ?

C.

1. Find the value of a and b so that $x^2+3xy+4y^2$ may exactly divide $x^6+7x^5y+6x^4y^2+5x^3y^3+ax^2y^4+bxy^5+12y^6$.

2. If $x^2+15x+m$ is exactly divisible by $x+7$, what is the value of m ?

3. The product of two factors is $(2x+3y)^3+(2y+3z)^3$, and one of them is $2x+5y+3z$; find the other.

4. Multiply x^2-x+1, x^2+x+1 and x^4-x^2+1 together, and divide $x^{16}+x^8+1$ by the product.

5. Divide the difference between $(2a+3b)^2$ and $(3a+2b)^2$ by $5(a+b)$.

6. Three factors of $x^4-4bx^3+6b^2x^2-4b^3x+b^4$ when multiplied together give $(x-b)^3$ as product, find the fourth factor.

7. Divide the product of $(a+b-c)$; $(a-b+c)$; $(b+c-a)$ by $a^2-b^2-c^2+2bc$.

8. Divide the product of $6x^2-17ax+12a^2$ and $4x-5a$ by $3x-4a$.

9. Divide $49x^2-16z^2+21xy+12yz$ by $7x+3y-4z$.

10. By what expression must a^2-bc be multiplied that the product may be $a^3+a^2b+a^2c-abc-b^2c-bc^2$?

EXERCISE VII.

MISCELLANEOUS EXERCISE.

1. The product of two expressions is $a^4+8a^3+24a^2+32a+16$ and one of them is a^2+4a+4; what is the other?

2. Multiply $(x-y)^2-xy$ by $(x+y)^2+xy$.

3. The sum of two expressions is $x-\frac{y}{2}-\frac{z}{3}$ and one is $\frac{1}{2}(x+y)+\frac{2z}{3}$ what is the other?

4. Find the continued product of $x-3$; $x-1$; $x+1$; $x+3$.

5. Find the divisor by which $36x-23x^2+12x^4+8$ must be divided so that the quotient will be $3x^2+6x+1$, and the remainder $2x+1$.

6. Divide $\frac{1}{4}-4a^4$ by $\frac{1}{2}+a$.

7. What expression must be added to

$$\frac{3x^2}{2}-\frac{7x}{2}-2 \text{ to make } x^2-2x-\frac{5}{3}?$$

8. By how much does $a(m+n)-b(m-p)-c(p-n)$ exceed $m(a-b)+p(b-c)+n(c-a)$?

9. What value of a will make the product of $3-8a$ and $3a+4$ equal to the product of $6a+11$ and $3-4a$?

10. Divide $\frac{a^3}{125}+\frac{b^3}{27}$ by $\frac{a}{5}+\frac{b}{3}$.

11. Show that the product of $8x^2+2ax-3a^2$ and $10x^2+10ax+5a^2$ may be written $(3x+a)^4-(x+2a)^4$.

12. Divide $z^4+1+\frac{1}{z^4}$ by $z^2-1+\frac{1}{z^2}$.

13. Add 1 to the continued product of x, $x+1$, $x+2$ and $x+3$, and divide the result by x^2+3x+1.

14. Find r and s in terms of a, b, p, q, so that $x^4+px^3+qx^2+rx+s$ may be divided by x^2+ax+b whatever x may be.

15. Divide $x^5-\frac{1}{x^5}$ by $x-\frac{1}{x}$.

16. From $x(a-b)-y(b-c)$ take $(a+b)x+(b+c)y$ and divide the difference by $x+y$.

17. Find the product of $(a+b)(a^2+ab+b^2)(a-b)(a^2-ab+b^2)$.

18. Determine a and b that in the product of x^2+x+1 and x^3+ax^2+bx+c the coefficients of x^4 and x^3 may vanish.

19. Multiply $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ by $\frac{1}{a}-\frac{1}{b}+\frac{1}{c}$.

20. Find the value of a and b so that $x^2-ax+12$ will divide $x^3-bx^2+2x+24$ without a remainder.

21. Divide $x^2-\left(a+\frac{1}{a}\right)x+1$ by $x-a$.

22. What value of m will make $6x^4-2x^3+2mx^2+2x+m$ exactly divisible by x^2-x+1?

23. What value must a, b, c, each have that x^3+ax^2+bx+c may have $x-1$, $x-2$, $x-3$, all as factors?

24. If $n^2=n+1$, prove that $x^4+ax^3+a^2x^2+a^3x+a^4$ is exactly divisible by $x^2+nax+a^2$.

25. Find the expression which divided by a^2+2a+4 will give a^3-8 for quotient.

26. By what expression must $x+3$ be multiplied to give x^7+2187?

27. Find the value of a which will make $x^4-x^3-x^2-ax$ divisible without a remainder by x^2+x.

28. Find the remainder when $5x^3-8x^2+8x+7$ is divided by $5x-3$.

29. If $4x^3+3x^2-18x+27$ multiplied by another expression is equal to $4x^5+11x^4+81$, find the other expression.

30. If $a=4$, $b=5$, by what must ax^2+bx+1 be multiplied to give $8x^3+(5a+2)x^2+(4b-3)x+3$ as product?

EXERCISE VIII.

HORNER'S METHOD OF DIVISION.

A.

1. $a^4+4a^2+16 \div a^2-2a+4$.
2. $5x^4-4x^3+3x^2+22x+55 \div x^2-3x+5$.
3. $x^5-5x^3+7x^2+6x+1 \div x^2+3x+1$.
4. $6x^7-x^6-11x^5+16x^4+x^3+8x^2-19x+20 \div 2x^3+x^2-3x+4$.
5. $6x^5+5x^4-17x^3-6x^2+10x-2 \div 3x^3-2x^2-4x+2$.
6. $x^5-x^4y+x^3y^2-x^2y^3+xy^4-y^5 \div x^3-y^3$.
7. $6x^5+7x^4+7x^3+5x^2+2x+1 \div 3x^3+2x^2+x+1$.
8. $x^6-2x^5-5x^4+20x^3-25x^2+14x-3 \div x^2+2x-3$.
9. $x^6-29x^5+55x^4-232x^3+351x^2-200 \div x^2+8$.
10. $21x^5-2x^4-70x^3-23x^2+33x+27 \div 7x^2+4x-9$.

B.

1. Find the remainder when x^3-3x^2+2x-7 is divided by $x-2$.

2. Find the remainder after dividing the following: $5x^4-6x^3+7x^2-3$ by $x+1$.

3. Find the remainder after dividing the following: $8x^3-16x^2-12x-10$ by $2x+5$.

4. Find the value of $5x^5-4x^4+3x^3-4x^2+x+4$ when $x=-4$.

5. Find the value of $x^6-102x^5+100x^4+102x^3-99x^2-201x$ when $x=101$.

6. Find the value of $7x^4-11x^2+x-50$ when $x=2$.

7. Find the value of $x^5-98x^4-98x^3-100x^2+98x+100$ when $x=99$.

8. Find the remainder when $8x^3+12x^2-4x-5$ is divided by $2x+3$.

9. Find the value of $5x^5-4x^4+3x^3-4x^2+x+4$ when $x=3$.

10. Find the remainder when $x^5-6x^4+5x^3-4x^2+3x-2$ is divided by $x+5$.

11. Find the value of $x^{10}-3x^7+x^4-5x+6$ when $x=1$.

12. Find the remainder when $(a+b+c)\ (ab+bc+ca)-abc$ is divided by $a+b$.

EXERCISE IX.

INVOLUTION.

Write down the square of the following:

1. $(a+b)$; $(2a+3b)$; $\left(\frac{x}{2}+\frac{y}{3}\right)$; $(a+2x)$.
2. $(ax+b)$; $(15x+14y)$; $\left(\frac{3a}{5x}+\frac{4x}{3a}\right)$; $\left(\frac{x}{y}+\frac{y}{x}\right)$.
3. $(a-b)$; $(a-2b)$; $(2x-6y)$; $\left(\frac{2x}{3y}-\frac{3y}{2x}\right)$.
4. $(7x^2-5y^2)$; $\left(\frac{x}{y}-\frac{y}{x}\right)$; $\left(\frac{x}{3}-\frac{y}{4}\right)$; $\left(\frac{3x^2}{4}-\frac{4y^2}{5}\right)$.
5. $(a+b+c)$; $(2x+3y+4z)$; $(1+x+x^2)$.
6. $(4a+5b+6c)$; $\left(\frac{x}{2}+\frac{y}{3}+\frac{z}{4}\right)$; $\left(1+\frac{1}{2}x+\frac{1}{3}x^2\right)$.
7. $(a+b-c)$; $(a-b+c)$; $(a-b-c)$.
8. $(x-\frac{1}{2}y+1)$; (x^2-5x+7); (x^2-ax-b).
9. $\left(\frac{x^2}{3}+x+\frac{1}{2}\right)$; $\left(3x^2-\frac{x}{3}-3\right)$; $\left(\frac{2m}{3n}-\frac{3n}{2m}\right)$.
10. $(ax+by+c^2)$; $\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)$.

EXERCISE X.

Write down the cubes of

1. $x+y$; $x-y$; $x+y+z$; $x+y-z$.

2. $m+\frac{1}{m}$; $m-\frac{1}{m}$; $\frac{m}{n}+\frac{n}{m}$.

3. $a-b+c$; $a-b-c$; $1+x+x^2$.

4. Simplify $(a+3b)^2+2(a+3b)\,(a-b)+(a-b)^2$

5. Simplify $(a+b+c)^2+(b+c)^2-2(b+c)\,(a+b+c)$.

6. Show that $(mx+ny)^2+(nx-my)^2=(m^2+n^2)\,(x^2+y^2)$.

7. " " $(ax+by)^2+(cx+dy)^2+(ay-bx)^2+(cy-dx)^2=(a^2+b^2+c^2+d^2)\,(x^2+y^2)$.

8. Simplify $(a+b)^2-(b+c)^2+(c+d)^2-(d+a)^2$.

9. " $(1-a^2)^3+(1+a^2)^3$.

10. " $(3x-4y+5z)^3-(5z-4y)^3-3(3x-4y+5z)^2\,(5z-4y)+3(5z-4y)^2\,(3x-4y+5z)$.

EXERCISE XI.

MISCELLANEOUS EXERCISE.

1. Prove that $(2a-b)^2+(2b-c)^2+(2c-a)^2+2(2a-b)\,(2b-c)+2(2a-b)\,(2c-a)+2(2b-c)\,(2c-a)-(a+b+c)^2=0$.

2. What will $a^3+b^3+c^3-3abc$ become if $a+b+c=0$?

3. If $x+\frac{1}{x}=p$, prove $x^3+\frac{1}{x^3}=p^3-3p$.

4. Complete the square in the following: $x^2+y^2+z^2+2xy+\quad+\quad$ etc.

5. Simplify $(1-x^2)^3+(1+x^2)^3$.

6. Simplify $(a+b-c)^3+3(a+b-c)^2c+c^3+3(a+b-c)c^2$.

7. If $x=2y+3z$, show that $x^3-8y^3-27z^3=18xyz$.

8. Simplify $(2a-3b)^3+(4b-5a)^3+(3a-b)^3-3(2a-3b)\,(4b-5a)\,(3a-b)$.

9. Simplify $(1+x+x^2)^3-(1-x+x^2)^3-6x(1+x^2+x^4)$.

10. Find the value of $x^6-y^6+z^6+3x^2y^2z^2$ when $x^2-y^2+z^2=0$.

11. Simplify $(a-b-c)^3+(b+c)^3+3(a-b-c)^2(b+c)+3(a-b-c)\,(b+c)^2$.

12. Simplify $(x^2+xy+y^2)^3+(x^2-xy+y^2)^3+6(x^2+y^2)\,(x^4+x^2y^2+y^4)$.

FACTORING.

EXERCISE XII.

Factor

1. $ax+bx+cdx$; $ax+ay+az+px+py+pz$.
2. $ax^2-ay^2-bx^2+by^2$; $1-a-b+ab$.
3. $9a^4b^2+33a^2b^3-12b^4$; $(1+x^2)(1-x^2)+q-px^2-qx^2+p$.
4. $15ab^2c^3+12a^3bc^2-21ac^4$; $2a^2x^2+1-2a^2-x^2$.
5. $4ax+ay+4bx+by$; $(a+1)(a-1)+ab+1-b-a$.
6. $2af+2bx+2ax+2bf$; $x^2+3y-3x-xy$.
7. $acx^2+adx-bcx-bd$; $(x^2-a^2)(x^2+a^2)+ax^3-a^3x$.
8. $1-a+b+p+q-ap+bp-aq+bq$.
9. $a-b+c-ab-bc+b^2$; a^3+a^2-a-1.
10. $ad+db+ce-ae+bf-cf+af-cd-be$.

EXERCISE XIII.

COMPLETE SQUARES.

A.

Factor

1. $a^2+8ab+16b^2$; $a^2+14ab+49b^2$.
2. $a^2+36a+324$; $x^2-10ax+25a^2$.
3. $x^2y^2-16xy+64$; $4bx^2-20bxy+25by^2$.
4. $m^4n^4+2m^2n^2+1$; $16x^4+16x^2+4$.
5. $a^2-18a+81$; $1-8x+16x^2$.
6. $\frac{1}{4}x^4+2x^2y+4y^2$; $c^{2m}-2c^m+1$.
7. $x^2y^4-12xy^2+36$; $(a+b)^2+2(a+b)(c+d)+(c+d)^2$.
8. $x^2+y^2+z^2+2xy+2xz+2yz$; $16x^4+72x^2y^2+81y^4$.
9. $9a^2+4b^2+16c^2-16bc+24ac-12ab$.
10. $9x^2+4y^2+z^2-4yz-6xz+12xy$.

B.

1. $\frac{1}{4}x^4+16y^2z^2-4x^2yz$; $a^2+b^2+c^2-2ab+2ac-2bc$.

2. $(a-b)^2+(b-c)^2+(c-a)^2+2(a-b)(b-c)+2(a-b)(c-a)+2(b-c)(c-a)$.

3. $4a^4-12a^2b+9b^2+16a^2c-24bc+16c^2$.

4. $\frac{4a^4}{9}+\frac{16b^4}{25}+\frac{9c^4}{16}+\frac{6b^2c^2}{5}+c^2a^2+\frac{16a^2b^2}{5}$.

5. $4b^2+9c^2-4b-6c+12bc+1$.

6. $x^2+\frac{y^2}{4}+\frac{z^2}{9}-\frac{yz}{3}+\frac{2xz}{3}-xy$.

7. $(p+q+r)^2-2s(p+q+r)+s^2$.

8. $\frac{a^2}{b^2}-2+\frac{b^2}{a^2}$; $(2x-3y)^2+(2x+3y)^2-2(4x^2-9y^2)$.

9. $4a^4-12a^3+25a^2-24a+16$.

10. $a^4+b^4+c^4+2a^2b^2-2a^2c^2-2b^2c^2$.

C.

1. $4(a+b)^2+12(a+b)(c+d)+9(c+d)^2$.

2. $4a^2+b^2+c^2-4ab+4ac-2bc$.

3. $256x^{2m}-448x^{m+n}+196x^{2n}$.

4. If $x^2+y^2=z^2$ and $xy+xz+yz=0$, show that $(x+y+z)^2=2z^2$.

5. Write down the square of $a^3-\frac{1}{2}a^2-6a+\frac{1}{3}$.

6. Arrange the six factors of $(6a^2+a-2)(3a^2-7a-6)(2a^2-7a+3)$ in the form of three squares.

7. Find the two equal factors of $\frac{a^2}{16}+9-4x+\frac{4x^2}{9}+\frac{3a}{2}-\frac{ax}{3}$.

8. Show that $2(x-y)(y-z)+2(y-z)(z-x)+2(z-x)(x-y)$ is the sum of three squares.

9. Find two equal factors of $(x^2-xy)^2-2(x^2-xy)(xy-y^2)+(xy-y^2)^2$.

10. Find two equal factors of $\frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{a^2}-2\frac{a}{c}-2\frac{c}{b}+2\frac{b}{a}$.

EXERCISE XIV.

DIFFERENCE OF SQUARES.

A.

1. $4x^2-9y^2$; $144x^2-289y^2$; $16x^4-1$.

2. $(2a-b)^2-c^2$; $(4x+y)^2-z^2$; $(3m+2n)^2-p^2$.

3. 199^2-1; $x^2-(y-z)^2$; $(a-2b)^2-(b-c)^2$.

4. $(x^2+y^2+z^2)^2-4x^2z^2$; $a^2-2ab+b^2-x^2-2xy-y^2$.

5. $4(ad+bc)^2-(a^2-b^2-c^2+d^2)^2$; $4a^2b^2-(a^2+b^2-c^2)^2$.

6. $x^{2n}-y^{2n}$; $(3x+5)^2-(5x+3)^2$; $(a+b+c+d)^2-(a-b+c-d)^2$.

7. $(x^2+y^2+z^2-xy-xz-yz)^2-(xy+yz+xz)^2$.

8. $15x^2-60y^2$; $243x^4-48y^4$; $1-4a^2b^{2n}$.

9. $(a-2b)^2-(2a-3b+4c)^2$; $a^4+2a^3+a^2-b^4+2b^3-b^2$.

10. $(x^2+y^2+z^2-2xy+2xz-2yz)-(y+z)^2$.

B.

1. Write down the quotient of
$(4x-3y-2z)^2-(3x-2y+3z)^2$ by $x-y-5z$.

2. Write down the value of $(x^2+y^2+z^2)(x^2+y^2-z^2)$.

3. Factor $a^4+b^4-c^4-d^4+2a^2b^2-2c^2d^2$.

4. Show that $(5x-3y-4)^2-(3x+7y+4)^2$ is exactly divisible by $2x+y$.

5. From $a^2-b^2=(a+b)(a-b)$, find the difference of the squares of $118\frac{1}{4}$ and $121\frac{3}{4}$.

6. Prove $\frac{1}{a^2}-\frac{2}{ab}+\frac{1}{b^2}-\frac{1}{c^2}=\left(\frac{1}{a}-\frac{1}{b}+\frac{1}{c}\right)\left(\frac{1}{a}-\frac{1}{b}-\frac{1}{c}\right)$.

7. Show that $1+\frac{a^2+b^2-c^2}{2ab}=\frac{(a+b+c)(a+b-c)}{2ab}$.

8. If $a+b+c+d=2s$, show that $4(ab+cd)^2-(a^2+b^2-c^2-d^2)^2 =16(s-a)(s-b)(s-c)(s-d)$.

9. Show that $(a^2+b^2+4ab)^2-(a^2+b^2)^2=8ab(a+b)^2$.

10. Show that $(x+y)^2-z^2+(y+z)^2-x^2+(z+x)^2-y^2=(x+y+z)^2$.

EXERCISE XV.

EXTENDED APPLICATION OF $(x \pm y)^2$ AND x^2-y^2.

A.

1. Factor x^4+4y^4; x^4-15x^2+9; x^4+x^2+1.

2. $256x^4+16x^2+1$; x^4+5x^2+49; $a^4+b^4-11a^2b^2$.

3. $4a^4-37a^2b^2+9b^4$; $9x^4+5x^2y^2+y^4$; $x^4+y^4-18x^2y^2$.

4. $m^4+n^4-18m^2n^2$; x^8+x^4+1; $c^4+c^2a^2+a^4$.

5. a^4+64b^4; $625a^4+25a^2+1$; $a^4-19a^2b^2+9b^4$.

6. $9a^4-4a^2b^2+\frac{b^4}{4}$; $4x^4-\frac{49x^2}{4}+9$.

7. $x^4 + \frac{16x^2}{9} + \frac{256}{81}$; $x^4 + 25x^2 + 625$.

8. $16a^4 - 17a^2b^2 + b^4$; $x^{4m} + 64y^{4m}$; $x^4 - 7x^2 + 1$.

9. $16m^4 - 28m^2n^2 + 9n^4$.

10. $(x+y)^4 - 7z^2(x+y)^2 + z^4$; $(a+b)^4 - 3c^2(a+b)^2 + c^4$.

B.

1. $9a^4 + 3a^2b^2 + 4b^4$; $x^4 + 7x^2 + 16$; $16x^4 + 36x^2y^2 + 81y^4$.

2. $\frac{a^4}{9} - \frac{5a^2b^2}{6} + \frac{b^4}{16}$; $\frac{a^4}{256} + \frac{a^2b^2}{144} + \frac{b^4}{81}$.

3. $a^4 + b^4 + c^4 - 2a^2b^2 - 2b^2c^2 - 2c^2a^2$.

4. $x^4 + 4(y+z)^4$; $(a+b)^4 + (a-b)^4 + (a^2-b^2)^2$.

5. $\frac{1}{a^4} + \frac{1}{x^4} + \frac{1}{a^2x^2}$; $\frac{9}{x^4} - \frac{3}{x^2y^2} + \frac{1}{y^4}$.

6. $4(a+b)^4 + 9(a-b)^4 - 21(a^2-b^2)^2$.

7. $16a^4 + 4(b-c)^4 - 9a^2(b-c)^2$.

8. $(x^2+y^2-xy)^4 - 7(x^3+y^3)^2 + (x+y)^4$.

9. $(a+b)^4 + 4(a-b)^4$; $4x^4 - 13x^2y^2 + 9y^4$.

10. $\frac{1}{a^4} + \frac{2}{a^2b^2} + \frac{9}{b^4}$; $\frac{16}{a^4} + \frac{1}{b^4} + \frac{4}{a^2b^2}$.

EXERCISE XVI.

TRINOMIALS.

A.

1. $x^2 + 8x + 12$; $x^2 + 9x + 20$; $x^2 + 47x + 370$.

2. $x^2 + 89x + 1960$; $x^2 - 27x + 182$; $x^2 - 19x - 150$.

3. $x^2 + 16x - 80$; $x^2 - 88x + 1612$; $x^2 - 37x - 120$.

4. $15x^2 + 17x + 4$; $6x^2 - 5xy - 6y^2$; $16c^2 - 16ac - 21a^2$.

5. $x^2 - \frac{9x}{20} - 1$; $x^2 - \frac{21x}{10} - 1$; $x^2 - \frac{35x}{18} + \frac{17}{18}$.

6. $x^2 + 33x + 252$; $x^2 - 92x - 693$; $x^2 - 37x - 528$.

7. $21x^2 - 55xy + 14y^2$; $x^4 - \frac{5x^2}{12} - \frac{1}{4}$; $x^2 - \frac{35x}{18} + \frac{2}{3}$.

8. $6(2x+3y)^2 + 5(6x^2 + 5xy - 6y^2) - 6(3x-2y)^2$.

9. $4(x+2)^4 - 37x^2(x+2)^2 + 9x^4$.

10. $(a-b)^{2m} - 44(a-b)^m + 363$.

B.

1. $72x^2-145x+72$; $8x^2-38x+35$.
2. $24x^2-29xy-4y^2$; $10x^2-17x+3$.
3. $15x^2+114x+99$; $12x^2+19x-21$.
4. $6a^2-ab-15b^2$; $32z^2-24xz-20x^2$.
5. $413x^2-606xy-299y^2$; $204x^2-329xy-200y^2$.
6. $45x^2+614x-22499$; $30x^2+859x+5247$.
7. $78x^2-4231x+48015$; $56x^2+137x-27885$.
8. $42x^2-135x-11877$; $96x^2+580x-9919$.
9. $34x^2-2001x-30745$.
10. $28a^2-411a-83467$.

EXERCISE XVII.

POLYNOMIALS.

A.

1. $20x^2+2xy-6y^2-8x+4y$.
2. $6a^2-7ab-20b^2-6a+15b$.
3. $7x-42y-2x^2+9xy+18y^2$.
4. $3x^2+19xy+20y^2+2x+10y$.
5. $x^2-xy-6y^2-4xz+12yz$.
6. $18x^2-24xy+8y^2+9xz-6yz$.
7. $6x^2-6y^2-20z^2+22yz+7xz-5xy$.
8. $55a^2+6b^2-12c^2+34bc-8ac-71ab$.
9. $6a^2-23ab+10ac-25bc+21b^2-4c^2$.
10. $3m^2+2mn-n^2-5r^2-6nr-2mr$.

B.

1. $7x^2-xy-6y^2-6x-20y-16$.
2. $20x^2-15xy-5y^2-68x-42y-88$.
3. $20x^2-20y^2+9xy+28x+35y$.
4. $x^2-xy-12y^2-5x-15y$.
5. $6x^2+6y^2-13xy-8z^2-2yz+8xz$.

6. $4a^2 - 15b^2 - 4ab - 21c^2 - 36bc - 8ac$.

7. $15x^2 - 18x - 28y^2 + 42y - 23xy$.

8. $5a^2 - 19ab + 12b^2 - 2a + 6b$.

9. $18x^2 - 42xy + 20y^2 + 9x - 15y$.

10. $20x^2 - 24x + 12y^2 - 18y + 31xy$.

C.

1. $4a^2 - 5ab - 21b^2 + 4a - 12b$.

2. $6x^2 - 7xy + 2xz - 20y^2 + 64yz - 48z^2$.

3. $2x^2 + 5xy + 2y^2 + 13xz + 17yz + 21z^2$.

4. $24x^2 + 37xy + 12x - 5y^2 + 20y$.

5. $3x^2 - xy + 4xz - 4y^2 - 3yz + z^2$.

6. $2x^2 + 5mx + 3m^2 + 2sx + 5ms - 12s^2$.

7. Show that $7m - 3n$ is a factor of $28m^2 + 21pm - 75mn - 9pn + 27n^2$ and write down the other factor.

8. If $x+p$ and $x+q$ are factors of $x^2 + (p+m)x + mp$ and $x^2 + (q+m)x + qm$ respectively, show that both are factors of $x^3 + (p+q+m)x^2 + (pq+pm+qm)x + pqm$.

9. Show that one of the factors of $6x^2 - 7xy + 14x - 20y^2 - 35y$ is also a factor of $8x^2 - 16xy - 10y^2$.

10. Show that the factors of $16x^2 - 46xy + 15y^2 + 76xz - 54yz + 48z^2$ may be written as the difference of two squares.

EXERCISE XVIII.

APPLICATION OF $x^3 \pm y^3$.

1. $a^3 + b^3$; $(a+x)^3 + y^3$; $(m+n)^3 + (p+q)^3$.

2. $(m^2 - mn + n^2)^3 + (m^2 + mn + n^2)^3$; $a^6 + b^6$.

3. $a^{12} + b^{12}$; $a^{15} + b^{15}$; $8a^3 + 27b^3$.

4. $x^{18} + y^9$; $125x^{21} + 512y^{24}$; $a^3 - (b+c)^3$.

5. $8x^3 - 64y^3$; $a^{24} - b^{33}$; $x^3 - 3ax^2 + 3a^2x - a^3 + b^3$.

6. Show without multiplying out that $(x^2 + xy + y^2)^3 - (x^2 - xy + y^2)^3$ is divisible by x and y.

7. Show that the sum of the cubes of $2x^2 - 5x - 9$ and $x^2 + 6x - 5$ is divisible by either $3x + 7$ or $x - 2$.

8. Write the quotient of $x^3+3x^2a+3xa^2+a^3+b^3$ by $x+a+b$.

9. Show that $(ax+by+cz)^3+(cx-by+az)^3$ is divisible by $(a+c)(x+z)$.

10. Show that $2(b+d)$ is a factor of $(a+b+c+d)^3-(a-b+c-d)^3$, and that $(2x^2+5x-9)^3-(x^2+6x-7)^3$ is divisible by $(x-2)(x+1)$.

EXERCISE XIX.

GENERAL EXERCISES IN FACTORING.

A.

1. $a(x+y)^2-bc(x+y)$; $15p^2+67p-24$.
2. $(a+b-c)^2-(a-b+c)^2$; $a^2-b^2-c^2-2a-2b-2c-2bc$.
3. $2x^2+11xy+12y^2+7xz+13yz+3z^2$; $x^{14}+3x^7y^7-4y^{14}$.
4. $(b-c)x^2+2(ab-ac)x+a^2b-a^2c$; $2(a+b)^2+5(a+b)+2$.
5. $x^4+x^3y-xy^3-y^4$; $a^3-b^3-3a^2+3a-1$.
6. $(ac+bd)^2+(ad-bc)^2$; $(x^2+5x)^2+10(x^2+5x)+24$.
7. $x^5+x^4y+x^3y^2+x^2y^3+xy^4+y^5$; x^4+x^3+x-1.
8. a^3-2a^2+2a-1; x^3+4x^2+5x+2; x^3+5x^2+7x+3.
9. $x^3+6x^2+11x+6$; $x^3-7x^2+14x-8$; $x^3-9x^2+26x-24$.
10. x^3+2x^2-5x-6; $x^3-x^2-22x+40$; x^3-2x^2-5x+6.

B.

1. $2x^3+11x^2+17x+6$; $3x^3+11x^2+12x+4$.
2. $3x^3+5x^2+7x+5$; $2x^3+7x^2+2x-3$.
3. $4x^3+8x^2-x-2$; $9x^3-45x^2-4x+20$.
4. $6x^3-11x^2-31x+30$; $a^3+6a^2b+12ab^2+8b^3$.

5. Prove that $(b-c)a^3+(c-a)b^3+(a-b)c^3$ is exactly divisible by $a+b+c$.

6. Express $x^4-px^3+qx^2-x^2+px-q$ in the form of three factors.

7. Factor $a(b+bc-c)+b(c+ca-a)+c(a+ab-b)$.

8. Express $(x+1)(x+2)(x+5)(x+10)-36x-136$ in two linear and one quadratic factor.

9. Resolve into three factors x^4-11x^2+10.

10. Resolve into three factors x^3-2x^2-x+2.

EXERCISE XX.

H. C. F.

A.

1. Find H. C. F. of $6(a-x)$; $4(a^2-x^2)$; $(a+b)^2$ and a^2-b^2.

2. ″ ″ $x^2-(a-b)x-ab$ and $x^2-(a+b)x+ab$.

3. ″ ″ $x^2-9x+14$ and $x^2-11x+28$.

4. ″ ″ $x^2-15x+36$ and $x^2-9x-36$.

5. ″ ″ x^4+x^2-6 and x^4-3x^2+2.

6. ″ ″ x^3-2x^2+3x-6 and $x^4-x^3-x^2-2x$.

7. ″ ″ $6x^2+17x+12$ and $10x^2+3x-18$.

8. ″ ″ $24x^4-22x^2+5$ and $48x^4+16x^2-15$.

9. ″ ″ $6a^3-6a^2+2a-2$ and $12a^2-15a+3$.

10. ″ ″ $3a^4+8a^3+4a^2$; $3a^5+11a^4+6a^3$; $3a^4-16a^3-12a^2$.

B.

Find H. C. F. of

1. x^2+x-30; $x^2+11x+30$; x^2-x-42.

2. $21x^2+8xy-4y^2$; $21x^2-20xy+4y^2$; $49x^2-28xy+4y^2$.

3. $x^3+x^2y-3xy^2+y^3$; $x^3+3x^2y+xy^2-y^3$.

4. $x^3-2x^2-15x+36$; $3x^2-4x-15$.

5. $3x^5-3x^4-53x^3-43x^2+34x+30$ and $3x^5+3x^4-53x^3+43x^2+34x-30$.

6. $x^3+3x^2y+3xy^2+y^3$; $x^3+x^2y+xy^2+y^3$; $x^3+x^2y-xy^2-y^3$.

7. Find H. C. F. of x^3-6x+9 and x^3+4x^2-9 and also what value of x will make both quantities vanish.

8. What value other than zero must be given a so that x^3-x-a and x^2+x-a may have a common factor?

9. Find H. C. F. of $2x^3+x^2-x-2$ and $x^5-x^3-2x^2+2x$ and show that its square is a factor of the latter expression.

10. Show that $(x-2)^2$ is a common factor of $x^7-6x^6+13x^5-12x^4+4x^3$ and $x^7-5x^6+8x^5-4x^4$, and find H. C. F.

EXERCISE XXI.

L. C. M.

Find L. C. M. of

1. $6x^2-2x$ and $9x^3-3x^2$.

2. $3x^2+11x+6$ and x^2+5x+6.

3. $x+2$; x^2-1; x^2+x-2.

4. x^2+5x+4; x^2+2x-8; $x^2+7x+12$.

5. a^2-1; a^2-9; $a^2+2a-15$.

6. $x^3+12x^2+47x+60$; $x^3+13x^2+56x+80$.

7. $x^3-6x^2+11x-6$ and $x^3-9x^2+26x-24$.

8. a^3-1; a^4-1; a^6-1.

9. $(x+y)^2$; $(x-y)^2$; x^3-y^3 and x^3+y^3.

10. $x^2+6x-27$; $x^2+17x+72$; x^2-64; $x^2-11x+24$.

EXERCISE XXII.

GENERAL EXERCISE—H. C. F. AND L. C. M.

1. Find by factoring the H. C. F. and L. C. M. of a^3-3a^2+a+1 and a^2-3a+2.

2. Of x^3-x^2-4x-4 and $x^3+6x^2+11x+6$.

3. Determine the algebraic expression which involves the lowest possible dimensions of x that can be exactly divided by x^2+5x+6 and x^2+7x+8.

4. If $x-3$ measures x^2-7x+a, find a.

5. If $x+4$ measures x^2-x-a, find a.

6. Find the values of a and b in order that $x-3$ may be a common factor of x^2-7x+a and x^2+x-b.

7. What value of a will make $x-7$ a measure of $x^2-ax+21$?

8. If $4x^2+bx-20$ is a multiple of $2x+5$, find b.

9. $x+3$ is a measure of $x^3-6x^2+11x-c$, find c.

10. If $x+9$ is a measure of x^2-a^2, find a.

11. The H. C. F. of two expressions is x and the L. C. M. abx, find the product of the two expressions.

12. If $x+4$ and $x+7$ are both measures of $x^2+11x+m$, find m.

13. The H. C. F. divided into L. C. M. gives $2ab$ for quotient. If H. C. F. is $2x$, find two sets of expressions that will satisfy the conditions.

14. If $x+4$ and $x+3$ measure $x^3+ax^2+26x+24$, find a, and also what other expression will be a measure of the given one.

15. Show that if a quantity divide A and B exactly it will divide $pA \pm qB$.

16. If x^2+mx+n is a measure of x^3+px^2+qx+r, prove that $nq-n^2=rm$.

17. Show that we can obtain L. C. M. of two algebraical expressions A and B by dividing their product by the H. C. F.

18. The H. C. F. of two expressions is $x+3$ and the L. C. M. $x^4+7x^3+12x^2-9x-27$, one expression is x^3+4x^2-9, find the other.

19. If a number be a measure of two others, prove that it will also be a measure of the difference of any multiples of these numbers.

20. If x^2-px+q is divisible by $x-r$ and $x-s$, show that $p=r+s$ and $q=rs$.

21. Find L. C. M. of $x^2-5x-14$; $x^2-4x-21$; and $x^3-3x^2-25x-21$, and for what value of x will all three expressions vanish.

22. Find H. C. F. of x^3-x^2-2x+2 and $x^4-3x^3+2x^2+x-1$. What value of x will make both expressions vanish ?

FRACTIONS.

EXERCISE XXIII.

Simplify the following:

1. $\dfrac{a^2-ab}{a^2b-abc+b^2c-ab^2}$; $\dfrac{3}{(x-2)(x-3)}-\dfrac{6}{(x-1)(x-2)(x-3)}$.

2. $\dfrac{x^2-(a+b)x+ab}{x^2+(c-a)x-ac}$; $\dfrac{a^2-9b^2}{a^2-2ab-15b^2}\div\dfrac{a^2+ab-12b^2}{a^2-5ab}$.

3. $\dfrac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-bc-ca-ab}$; $\dfrac{x^2-ax}{x^2-4}\times\dfrac{x^2+2x+ax+2a}{x^2-a^2}$.

4. $\dfrac{(y+z-2x)^2-(z+x-2y)^2}{(x+y+z)^2-(x+y-5z)^2}$; $\dfrac{b}{c}\left\{\dfrac{b}{c}-\dfrac{1}{bc}(b^2-c^2)\right\}$.

5. $\dfrac{(3x-2y)^2-(2x+2y)^2}{2x-8y}$; $\dfrac{x^2+x-6}{x^2+2x-8}\times\dfrac{x^2+4x}{x^2-9}$.

6. $\dfrac{a^4-a^3-a+1}{a^4-2a^3-a^2-2a+1}$; $\dfrac{a^3-3a^2b+3ab^2-2b^3}{a^4+a^2b^2+b^4}$.

7. $\dfrac{1}{x+3y}+\dfrac{6y}{x^2-9y^2}-\dfrac{1}{3y-x}$.

8. $\dfrac{a-3}{a-2}-\dfrac{a-3}{a-1}-\dfrac{1+3a}{a^2-3a+2}$.

9. $\dfrac{1}{x^2-4x+3}-\dfrac{1}{x^2-3x+2}-\dfrac{1}{x^2-5x+6}$.

10. $\dfrac{2x+10}{8}+\dfrac{3x+1}{7}-\dfrac{9x+13}{4}+\dfrac{247-8x}{14}$.

EXERCISE XXIV.

Simplify

1. $\dfrac{3x+2}{(x-1)^2}-\dfrac{6}{x^2-1}-\dfrac{3x-2}{(x+1)^2}$.

2. $\dfrac{a^3-x^3}{ax}\left(\dfrac{a+x}{a^2-x^2}-\dfrac{a-x}{a^2+ax+x^2}\right)$.

3. $\dfrac{12x^2+x-1}{1-8x+16x^2}\div\dfrac{1+6x+9x^2}{16x^2-1}$.

4. $\frac{x^2-7xy+12y^2}{x^2+5xy+6y^2} \div \frac{x^2-5xy+4y^2}{x^2+xy-2y^2}$.

5. $\frac{2a}{(x-2a)^2} - \frac{x-a}{x^2-5ax+6a^2} + \frac{2}{x-3a}$.

6. $\frac{x^2-6x+8}{x^2-4x+3} \times \frac{x^2-5x+6}{x^2-2x-8} \div \frac{(x-2)^2}{x^2-1}$.

7. $\frac{1+x}{1+x+x^2} + \frac{1-x}{1-x+x^2} - \frac{2}{1+x^2+x^4}$.

8. $\left(2a+3x-\frac{24ax}{2a+3x}\right)\left(2a-3x+\frac{24ax}{2a-3x}\right)$.

9. $\frac{a(a^2-ab+b^2)}{a-b} \times \frac{a^2-b^2}{a^3+b^3} \times \frac{2a^2+2b^2}{a^2-b^2} \times \frac{a^2+b^2}{a^2-b^2}$.

10. $\left(\frac{2x}{x+y}+\frac{y}{x-y}-\frac{y^2}{x^2-y^2}\right) \div \left(\frac{1}{x+y}+\frac{x}{x^2-y^2}\right)$.

EXERCISE XXV.

Simplify

1. $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$; $\frac{a-b}{ab}+\frac{b-c}{bc}+\frac{c-a}{ac}$.

2. $\frac{a}{a-x}-\frac{x}{a+x}$; $\frac{3x+2y}{3x-2y}-\frac{3x-2y}{3x+2y}$.

3. $\frac{x+1}{x^2+x+1}+\frac{x-1}{x^2-x+1}$; $\frac{1}{a+x}+\frac{1}{a-x}-\frac{2a}{a^2-x^2}$.

4. $\frac{1}{a+b}+\frac{1}{a-b}+\frac{2b}{a^2-b^2}$.

5. $\frac{a+b}{a}+\frac{a}{a^2-b^2}+\frac{b}{a-b}+\frac{a-b}{a+b}$.

6. $\frac{1}{(a-b)(a-c)}+\frac{1}{(b-a)(b-c)}+\frac{1}{(c-a)(c-b)}$.

7. $\frac{(x-a)(x-3)}{(x+4)^2} \times \frac{x+4}{x-a}$.

8. $\frac{2x^2-8x+6}{3x^2-15x+12} \times \frac{3x^2-27x+60}{2x^2-10x} \div \frac{x^2-10x+21}{x^2-7x}$.

9. $\frac{1}{1+\frac{1}{a}} \times \frac{1}{1-\frac{1}{a}} \div \frac{1}{a-\frac{1}{a}}$.

10. $\frac{6a^2-ab-12b^2}{6a^2+23ab+20b^2} \div \frac{12a^2-16ab-3b^2}{6a^2+7ab-20b^2}$.

EXERCISE XXVI.

1. $\left(\frac{1}{1+x}+\frac{x}{1-x}\right)\div\left(\frac{1}{1-x}-\frac{x}{1+x}\right).$

2. $\dfrac{\frac{x}{a}+\frac{x}{b}}{\frac{x}{a}-\frac{x}{b}}$; $\dfrac{\frac{2a+3}{3a+4}-\frac{4a+5}{5a+6}}{\frac{a+2}{2a+3}-\frac{3a+4}{4a+5}}.$

3. $\dfrac{\frac{a}{b}}{\frac{c}{d}}\times\dfrac{\frac{c}{d}}{\frac{a}{b}}$; $\left(\frac{2x}{x+y}+\frac{y}{x-y}-\frac{y^2}{x^2-y^2}\right)\div\left(\frac{1}{x+y}+\frac{x}{x^2-y^2}\right).$

4. $\frac{4a^4}{a^4-x^4}+\frac{2a^2}{x^2+a^2}+\frac{a}{a+x}+\frac{a}{a-x}.$

5. $\frac{a^2}{(a-b)\,(a-c)}+\frac{b^2}{(b-c)\ \,(b-a)}+\frac{c^2}{(c-a)\ \,(c-b)}.$

6. $\left(\frac{x^2}{y^2}+1+\frac{y^2}{x^2}\right)\left(\frac{x}{y}-\frac{y}{x}\right).$

7. $\left\{\frac{a+b}{a-b}+\frac{a-b}{a+b}\right\}\div\left\{\frac{a+b}{a-b}-\frac{a-b}{a+b}\right\}.$

8. $\frac{x}{x-a}-\frac{x}{x+a}-\left\{\frac{x+a}{x-a}-\frac{x-a}{x+a}\right\}\div\left\{\frac{x+a}{x-a}+\frac{x-a}{x+a}\right\}.$

9. $\left\{\frac{x-y}{2(x+y)}-\frac{x+y}{2(x-y)}-\frac{2y^2}{x^2-y^2}\right\}\frac{y-x}{2y}.$

10. $\frac{x^4+a^2x^2+a^4}{x^2-a^2}\times\frac{x+a}{x^2+ax+a^2}\div\frac{x^2-ax+a^2}{x-a}-\frac{x^3-a^3}{x^3+a^3}\times\frac{x^3-ax+a^2}{x-a}\times$
$\frac{x+a}{x^2+ax+a^2}.$

EQUATIONS.

EXERCISE XXVII.

A.

Solve the following equations:

1. $5x-(3x-7)=35-2x.$

2. $9x-3(5x-6)=-72.$

3. $(2x-7)(x+5)=2x(x-8)-x+265.$

4. $5(a+x)-5x=3(a+6x).$

5. $8(x-1)+17(x-3)=16x-32.$

6. $2x+34-30x+95=3x-119.$

7. $5x-17+3x-5=6x-7-8x+115.$

8. $\frac{x+3}{2}=\frac{x-2}{3}+\frac{3x-5}{12}+\frac{1}{4}.$

9. $\frac{7x+5}{23}+\frac{9x-1}{10}-\frac{x-9}{5}+\frac{2x-3}{15}=\frac{70}{3}.$

10. $\frac{4x+1}{5}-\frac{5x-3}{7}=\frac{7x-4}{12}-\frac{6x-5}{13}.$

B.

1. $\frac{5x-1}{2}-\frac{7x-2}{10}=\frac{66-5x}{10}.$

2. $4(x-3)-7(x-4)=6-x.$

3. $\frac{1}{8}(5x-10)-\frac{1}{12}(12x-13)=4x+\frac{1}{6}(2x-7).$

4. $\frac{7x+1}{x-1}=\frac{35}{9}\left(\frac{x+4}{x+2}\right)+3\frac{1}{9}.$

5. $(x-3)-3(x-5)+5(x-7)=0.$

6. $(x-5)(x-3)-(x-5)-(x+7)(x-2)=0.$

7. $\frac{x+1}{7}+x(x-2)=(x-1)^2.$

8. $\frac{7x+8}{8}-\frac{9x-12}{16}=\frac{9x+1}{10}-\frac{29-8x}{20}$.

9. $\frac{x}{2}-\frac{x-2}{3}=\frac{1}{4}\left\{x-\frac{2}{3}\left(\frac{5}{2}-x\right)\right\}-\frac{x-5}{3}$.

10. $\frac{4x-17}{9}-\frac{11-66x}{99}=6\frac{(54+x)}{54}$.

C.

1. $\frac{6x+13}{15}-\frac{3x+5}{5x-25}=\frac{2x}{5}$.

2. $\frac{6x+1}{15}-\frac{2x-4}{7x-16}=\frac{2x-1}{5}$.

3. $\frac{3x+1}{4}=\frac{9x+8}{12}-\frac{x+1}{x+8}$.

4. $\frac{9x+20}{36}=\frac{4x-12}{5x-4}+\frac{x}{4}$.

5. $\frac{10x+17}{18}-\frac{12x+2}{13x-16}=\frac{5x-4}{9}$.

6. $\frac{9x+5}{14}+\frac{8x-7}{6x+2}=\frac{36x+15}{56}+\frac{10\frac{1}{4}}{14}$

7. $\frac{6x+7}{15}-\frac{2x-2}{7x-6}=\frac{2x+1}{5}$.

8. $\frac{2x+3}{x+1}=\frac{4x+5}{4x+4}+\frac{3x+3}{3x+1}$.

9. $\frac{7x-4}{x-1}=\frac{7x-26}{x-3}$.

10. $\frac{x-1}{x-2}=\frac{7x-21}{7x-26}$.

D.

1. $\frac{4x+17}{x+3}+\frac{3x-10}{x-4}=7$.

2. $\frac{6x+8}{2x+1}-\frac{2x+38}{x+12}=1$.

3. $\frac{x-4}{x-5}-\frac{x-5}{x-6}=\frac{x-7}{x-8}-\frac{x-8}{x-9}$.

4. $\frac{x-1}{x-2}-\frac{x-2}{x-3}=\frac{x-4}{x-5}-\frac{x-5}{x-6}$.

5. $\frac{3x-1}{x-4}+\frac{2x-1}{x+4}=5+\frac{96}{x^2-16}$.

6. $\frac{6x-3}{3x-8}-\frac{x-1}{x-4}-1=0.$

7. $\frac{x-a}{x-b}+\frac{x-b}{x-c}=2.$

8. $\frac{x-1}{x+3}+\frac{x-3}{x+1}=2.$

9. $\frac{4x-3}{2x-1}=\frac{4x-7}{2x-5}.$

10. $\frac{x+10}{x+8}+\frac{x+1}{x+3}=\frac{x+11}{x+9}+\frac{x+2}{x+4}.$

E.

1. $\frac{3x^2+4x+3}{3x+4}=\frac{2x^2+8x+3}{2x+8}.$

2. $\frac{3+x}{1+x}+\frac{x+5}{x+7}=1+\frac{x^2-1}{7+8x+x^2}.$

3. $(a+x)\ (b+x)=(c+x)\ (d+x).$

4. $\frac{a}{x}+\frac{b}{c}=\frac{d}{c}\ ;\quad \frac{x}{a-b}=\frac{x}{a+b}+1.$

5. $\frac{x}{ab}+\frac{x}{bc}+\frac{x}{ca}=a+b+c.$

6. $\frac{a}{bx}+\frac{b}{ax}=a^2+b^2\ ;\quad \frac{x+a}{a-b}-\frac{x+a}{a+b}=\frac{2ax}{a^2-b^2}.$

7. $\frac{a}{x}-\frac{b}{a}=\frac{a+b}{x}+\frac{a^2-b^2}{a^2+ab}.$

8. $\frac{(m+n)\ (n-x)}{m-n}+x=0.$

9. $\frac{20x+11a}{25a}+\frac{5x+20a}{9x-16a}=\frac{4x}{5a}+\frac{61}{25}.$

10. $\frac{x+m}{x^2+mx+m^2}-\frac{x-m}{x^2-mx+m^2}=\frac{m^4}{x(x^4+m^2x^2+m^4)}.$

F.

1. $\frac{x-1}{2}+\frac{x-2}{3}+\frac{x-3}{4}=10.$

2. $3x-4\{9-(2x+7)+3x\}=13.$

3. $\frac{x}{3}-\frac{x-4}{5}-\frac{30-x}{6}=\frac{x}{4}-\frac{12-x}{3}-7.$

4. $(2x-3)^2-(4x^2-28x+49)=5x+15.$

5. $\frac{1}{2}x+\frac{1}{3}(2-x)=\frac{1}{4}\left\{2x-\frac{1}{3}(5+x)\right\}-\frac{x-5}{3}$.

6. $\frac{1}{4}(x-1)+\frac{18x-3}{35}=\frac{1}{5}(2x+7)+\frac{x+8}{7}$.

7. $783-5(9x+3)+12x+42-48x=0$.

8. $\frac{x-7}{11}-\frac{3x-5}{7}+1\frac{48}{77}=2(x-8)-1$.

9. $(3x+1)^2+(4x-2)^2=(5x-3)^2$.

10. $\frac{6}{x+10}-\frac{4x+44}{x^2+22x+120}=\frac{3}{x+12}$.

EXERCISE XXVIII.

PROBLEMS.

1. A man rides from London to Ingersoll at 8 miles per hour, and returns by Thamesford, 5 miles farther, at 9 miles per hour in 15 minutes more. How far from London to Ingersoll ?

2. A horse and buggy cost $280, and 5 times the price of the horse was equal to 9 times the price of the buggy. Find the price of each.

3. Divide $1496 between A, B, C so that B will get 3 times A's share, and C 4 times B's share.

4. A passenger leaves Montreal for Toronto, a distance of 333 miles, at the same time as one leaves Toronto for Montreal. The train from Montreal runs 42 miles per hour, and the one from Toronto 32 miles per hour. How long before they meet ?

5. A received a legacy of $4000, and B $3600 ; after each had invested an equal amount A had as much again as B. What did each invest ?

6. A field is 3 times as long as it is wide and contains 9600 sq. yds. less than another field which is the same length, but 40 yds. wider. Find the size of first field.

7. A has 3 times as much money as B. He spends $47.50 and then has only twice as much. How much had each at first ?

8. I bought 23 cows, some at $38 each and the rest at $50 each. The whole amount paid was $970. How many of each did I buy ?

9. A merchant's selling price is 20% advance on cost, but he gives a customer 10% off for cash and his profit on the article is then $1.20. Find the cost price.

10. A man gives to a beggar \$1.00 less than $\frac{1}{4}$ of his money, and then there remains \$1.00 less than $\frac{4}{5}$ of it. How much did he give the beggar?

11. A speculator owns \$5000 stock, some at 3%, 4 times as much at $3\frac{1}{2}$%, and the rest at 4%. Find the amount of each kind, when his income is \$176.00.

12. A man bought 80 yds. cloth, some at 50c. and some at 75c. per yd. He finds by selling all at 75c. per yd. he would gain \$2.50 more than by adding $12\frac{1}{2}$c. per yd. to the price of each. How much did he buy at 50c.?

13. A father gave his boy a certain sum of money every Monday. During the week he spent $\frac{1}{2}$ of all he had at the beginning, and at the end of the 3rd week he had \$1.40. What was his weekly allowance?

14. My income, \$960, is derived from money invested, some at 3% and some at 9%, but if the rates were interchanged my income would be doubled. How much is invested at 3%?

15. Two men begin business with equal capital. The first year one gains \$250 and the other's capital is reduced $\frac{1}{3}$, and the first had then twice as much as the second. How much had each at first?

16. A man sold to A $\frac{1}{3}$ of his cattle, $\frac{1}{6}$ to B, $\frac{1}{5}$ to C, and the rest, 27, to D. How many had he at first?

17. Divide 90 into 4 parts so that if the first be increased by 2, the second diminished by 2, the third multiplied by 2, and the fourth divided by 2, they shall be all equal.

18. A farmer sold a number of bags of wheat for \$72, and a second lot of 5 bags less at the same rate for \$63. Find the number of bags in each load.

19. Divide \$7400 among A, B, C so that A shall have \$120 more than B, and C \$106 less than A.

20. There is a number of two digits whose sum is 14, the unit's digit is the greater and $\frac{3}{17}$ of the number is half as much again as the unit's digit. What is the number?

21. A merchant increases his capital every year by $\frac{1}{3}$ of itself, but spends \$1000 for expenses. At the end of the 3rd year, after deducting \$1000 for expenses, his capital is doubled. What was it at first?

22. Divide 192 into two parts so that the larger divided by 7 may be 4 less than the smaller multiplied by 3.

23. A man divides a sum of money among A, B, C, giving A \$120 less than $\frac{1}{2}$ of it, B \$40 less than $\frac{1}{3}$, and C \$32 more than $\frac{1}{4}$ part. What did each get?

24. A man receives \$60000. He invests part in a house, $\frac{1}{3}$ of the remainder at 4%, and the rest at 5%. His income is \$1960. Find the cost of the house ?

25. A house and garden cost \$3400, and 5 times the price of the house equals 12 times the price of the garden. Find the price of each.

26. A farmer has horses worth \$62.50 each, sheep worth \$11.25. The total number of animals is 35, and the value \$957.50. Find the number of horses?

27. A person mixes tea at 60c. per lb. with some at \$1.00 per lb. He wishes to sell the mixture at $73\frac{1}{2}$c. per lb. and gain 10% on every lb. sold. How many lbs. of the inferior must he mix with each lb. of the superior ?

28. Divide 150 into two parts so that one part divided by 23 and the other by 27 will give 6 when added together.

29. The stones which pave a square court would cover a plot 6 yds. longer and width 4 yds. shorter than the side of the square. Find the area of the square.

30. A after spending \$50 less than $\frac{1}{3}$ of his income found that he had \$225 more than $\frac{1}{2}$ of it left. Find his income.

31. The left hand digit of a number consisting of two digits exceeds the right hand one by 4, and when the number is divided by the sum of the digits the quotient is 7. Find the number.

32. The length of a floor exceeds its breadth by 6 feet, but when each is increased 1 foot the area of the floor is increased by 31 sq. ft. Find the original dimensions.

33. Divide 40 into two parts so that 3 times one part and 5 times the other shall be 168.

34. The sum of \$380 was raised by A, B and C together. B gave \$50 more than A, and C as much as A and B. What did each give ?

35. The price of a work of several volumes is \$13.60, but if each volume cost 26c. more than it does the price would be \$16.20. How many volumes are there ?

36. The sum of \$2500 was divided among 4 societies so that the first and second together got \$1400 ; the first and third \$1300 ; the first and fourth \$1100. Find the share of each.

37. A general, after battle, found only $\frac{1}{2}$ of his army and 3600 more fit for duty, $\frac{1}{8}$ and 600 more wounded and the rest, $\frac{1}{5}$ of the whole, slain or prisoners. Find the number in his army.

EXERCISE XXIX.

1. $3x-7y=7$
$11x+5y=87.$

2. $x+y=10$
$x-y=4.$

3. $x+y=10$
$2x-3y=5.$

4. $4x-y=7$
$3x+4y=29.$

5. $3x-11y=4$
$5x-12y=13.$

6. $\frac{x}{3}+\frac{y}{8}=41.$
$4y-3x=0.$

7. $\frac{x}{3}+\frac{y}{4}=11$
$\frac{x}{5}+\frac{y}{6}=7.$

8. $8x-7y=12$
$\frac{x-2y}{4}+\frac{2x-y}{3}=1.$

9. $\frac{2}{x}+\frac{3}{y}=31$
$\frac{5}{y}-\frac{7}{x}=31.$

10. $\frac{x}{5}+\frac{y}{5}=8$
$\frac{x}{9}-\frac{y}{5}=0.$

11. $\frac{x}{5}+\frac{y}{6}=18$
$\frac{x}{2}-\frac{y}{4}=21.$

12. $\frac{x}{3}+\frac{y}{4}=6$
$8-\frac{x-y}{4}=7.$

13. $\frac{x+y}{2}=a$
$\frac{x-y}{2}=b.$

14. $x+y=m$
$ax+y=n.$

15. $\frac{m}{x}+\frac{n}{y}=a.$
$\frac{n}{x}+\frac{m}{y}=b$

16. $x+y=c$
$ax-by=0.$

17. $\frac{x}{2}+\frac{2}{y}=\frac{5}{4}$
$\frac{x}{3}+\frac{3}{y}=\frac{5}{3}.$

18. $7x-11y-3=0$
$5y-6x+7=0.$

19. $x+y-z=0$
$x-y+z=4$
$5x+y+z=20.$

20. $x+y+z=0$
$x+2y+3z=1$
$x+3y+4z=2.$

21. $x+6y+5z=0$
$2x-9y+3z=0$
$x+3y+z=3.$

22. $x+2y+3z=14$
$2x+3y+z=11$
$3x+y+2z=11.$

23. $7x+8z=53$
$9z-5x=21$
$12x-5y=11.$

24. $\frac{x}{5}-\frac{y}{3}=0$
$\frac{x}{2}+\frac{z}{5}=5$
$x-y+z=4.$

25. $\frac{1}{x}+\frac{1}{y}=5$
$\frac{1}{y}+\frac{1}{z}=8$
$\frac{1}{x}+\frac{1}{z}=7.$

26. $\frac{1}{x}+\frac{1}{y}-\frac{1}{z}=0$
$\frac{1}{x}-\frac{1}{y}+\frac{1}{z}=4$
$\frac{5}{x}+\frac{1}{y}+\frac{1}{z}=20.$

27. $x+y=3$
$y+z=5$
$z+p=7$
$x+p=5.$

28. $x+y+z=29.25$
$x+y-z=18.25$
$x-y+z=13.75.$

29. $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=9.$
$\frac{a}{x}+\frac{b}{y}-\frac{c}{z}=1.$
$\frac{7a}{x}-\frac{2b}{y}-\frac{c}{z}=4.$

30. $x+y+z=a.$
$x+y-z=b.$
$x-y-z=c.$

EXERCISE XXX.

1. A man bought two pieces of cloth for $50.60, one piece at $1.60 and the other at $1.80 per yd.; he sold them at an advance of 40c. per yd. and gained $12.00. Find the length of each piece.

2. Twenty-eight tons of coal are to be carried in carts and wagons. It is found it will require 15 carts and 12 wagons, or 24 carts and 8 wagons. How much can each cart and each wagon carry?

3. A certain number of two digits is equal to five times the sum of the digits, and if 9 be added to the number the digits are reversed. Find the number.

4. A person buys 8 lbs. tea and 3 lbs. sugar for $5.28, and 5 lbs. tea and 4 lbs. sugar for $3.64. Find price of each per lb.

5. A farmer sold to one man 30 bus. wheat and 40 bus. barley for $54.00, and to another 50 bus. wheat and 30 bus. barley for $68.00. Find price of each per bushel.

6. An account of $105 was paid in $5 bills and silver dollars; and 4 times the number of bills exceeded twice the number of silver dollars by 14. How many were there of each?

7. Two men, A and B, received $23.40 for work done. A worked 15 days and B 14 days. A received for 4 days $2.20 more than B did for 3 days. Find the daily wages of each.

8. A farmer sells to A 9 horses and 7 cows for $1200 and to B, at same price, 6 horses, 7 cows for the same amount. Find the price of each.

9. Find two numbers so that $\frac{1}{2}$ the first and $\frac{1}{3}$ the second shall be 9, and $\frac{1}{4}$ the first and $\frac{1}{5}$ the second shall be 5.

10. Two purses contain together $300, and if $30 are taken from one and put into the other there will be the same in each. How many dollars in each purse?

11. Find three numbers such that the sum of 1st and 2nd $=7$, 1st and 3rd $=8$, 2nd and 3rd $=9$.

12. There are three numbers such that the 1st and $\frac{1}{2}$ the second $=14$, the second with $\frac{1}{3}$ of the third $=18$, and the third with $\frac{1}{4}$ the first $=20$. Find the numbers.

13. A and B work together for 50 days at $1.20 each per day. A spent 12c. per day less than B, and at end of the time had saved twice as much as B, and the expense of two days over. What did each spend per day?

14. Find two numbers such that the sum of 7 times the greater and 5 times the less may be 332 and 51 times their difference 408.

15. If John gives Tom $10.00, Tom will have three times as much as John. If Tom gives John $10.00, John will have twice as much as Tom. What has each?

16. The sum of two numbers divided by 2 gives 24 for quotient, and the difference divided by 2 gives 17. What are the numbers?

17. The cost of 7 lbs. tea and 5 lbs. coffee is $7.04, and 4 lbs. tea and 9 lbs. coffee $6.48. What is the cost of 1 lb. of each?

18. A farmer bought 100 acres of land for $4220, part at $37 and part at $45 per acre. How many acres had he of each kind?

19. The sum of two digits composing a number is 6, and if the number is divided by the sum of the digits the quotient is 4. What is the number?

20. A certain fraction becomes 2 when 7 is added to the numerator, and 1 when 1 is subtracted from the denominator. What is the fraction?

21. A grocer bought tea at $2.00 per lb. and coffee at 50c. for $125. He sold the tea at $1.60 and coffee at 90c. per lb., and gained $20. How many lbs. of each did he buy?

22. What fraction is that to the numerator of which if 7 be added its value is $\frac{2}{3}$, but if 7 be taken from the denominator its value is $\frac{3}{8}$?

23. Three times the greater of two numbers exceeds twice the less by 115, and twice the greater together with 3 times the less is 250. Find the numbers.

24. If B gives A \$50 they will have equal sums, but if A gives B \$44 B's money will be equal to twice A's money. How much has each?

25. If A gives B \$5 he will have \$6 less than B, but if B gives A \$5, then 3 times A's money will be \$20 more than 4 times B's. How much has each?

26. A fraction becomes equal to $\frac{1}{2}$ when 3 is added to the numerator, and $\frac{2}{7}$ when 3 is added to the denominator. Determine the fraction.

27. A fraction becomes equal to $\frac{7}{9}$ when the denominator is increased by 4, and equal to $\frac{20}{41}$ when the numerator is decreased by 15. What is the fraction?

28. Find two fractions whose numerators are 2 and 5 respectively, and whose sum is $1\frac{1}{2}$; and if the denominators are interchanged the sum will be 2.

29. The sum of the two digits of a number is 8, and if 36 be added to the number the digits will be reversed. What is the number?

30. If a certain number be divided by the sum of its two digits, the quotient is 6 and remainder 3; if the digits be reversed and the resulting number divided by the sum of the digits, the quotient will be 4 with a remainder of 9. What is the number?

31. The first digit of a number when doubled is 3 more than the second, and the number itself is 6 less than 5 times the sum of the digits. What is the number?

32. A boatman rows 30 miles and back in 12 hours. He finds he can row 5 miles with the stream in the same time as he can row 3 against it. Find the time going up and down respectively.

33. A man has \$10000 invested, part at 5% and the rest at 4%. The income from the former is \$50 more than from the latter. How much has he in each investment?

34. A number consisting of two digits is equal to 7 times its unit's figure, and if the digits be reversed its value is increased by 18. What is the number?

35. A man travelled 240 miles in 4 days, diminishing his rate each day by the same distance; during the first two days he travelled 136 miles. How far did he go each day?

36. A and B engaged in trade, A with \$1100 and B \$1200. A lost half as much again as B, and B had then left half as much again as A. How much did each lose?

37. Divide 80 and 90 into two such parts so that the sum of one from each pair may be 100 and difference 30.

38. A farmer bought sheep at \$4 each and found he was \$8 short of money to pay for them, but had he only given \$3 each he would have had \$4 over. How many sheep were there, and how much money had he?

39. Find three numbers such that the 1st with $\frac{1}{2}$ the other two $=34$, the 2nd with $\frac{1}{3}$ the other two $=34$, and the 3rd with $\frac{1}{4}$ the other two $=34$.

40. A man has two kinds of coin; 12 of the first and 8 of the second are worth £$2\frac{3}{10}$, and 5 of the first and 10 of the second are worth £$1\frac{5}{8}$. Find the value of each kind of coin.

41. There are three numbers whose sum is 16. The third is 1 less than three times the first, and the second equal to the difference between the 1st and 3rd. Find the numbers.

42. The sum of \$960 is lent to A and B so that 4 times A's interest at 5% and 7 times B's interest at 4% is \$240. Find what each had.

43. The sum of the heights of two towers is 5 times their difference, and half the height of the higher one is 4 feet less than $\frac{4}{5}$ the height of the lower one. Find the height of each.

44. Sold 6 lbs. tea and 5 lbs. coffee for \$4.88, and 10 lbs. tea and 12 lbs. coffee for \$9.60. Find price of tea and coffee per lb.

MISCELLANEOUS EXERCISES.

A.

1. Divide $x^4+24x+55$ by x^2+4x+5.

2. What is the value of $a^3+b^3+c^3-3abc$ if $a=\ -1$ $b=3$ and $c=-2$?

3. Multiply x^2-4 by x^2-4x and divide the product by x^2+2x.

4. From x^3+11x take $6x^2+6$ and divide the remainder by $x-2$.

5. Add the product of $3x-2y$ and $2y-5x$ to the quotient of $28xy^3+105x^3y$ by $7xy$.

6. Subtract the square of $a+b$ from the square of $a-b$.

7. Divide the sum of $(x+y)\ (x+z)$ and $(x-y)\ (z-x)$ by $z+y$.

8. Factor $a^2-2a+ab-2b$.

9. Write down the product of x^3-9 and x^3+17.

10. Find the product of m^2-mn+n^2 and $m+n$.

11. What number is that from the double of which if 17 be subtracted the remainder is 69?

12. Simplify $2x^2+4xy-3y^2-(x-2y)^2$.

13. Factor $(a^2+ab+b^2)^2-(a^2-b^2)^2$.

14. Factor $3x^2+2xy$; $15a^2b-10b^2+15bc$.

15. Find H.C.F. of $x^{13}-y^{13}$ and x^5-y^5.

16. Find the co-efficient of x in $(x-5)\ (x-6)\ (x+7)$.

17. Multiply $a+b-\frac{1}{c}$ by $a-b-\frac{1}{c}$.

18. Factor $4a^4bc-3ab^2c+2abc^4$ making one factor a monominal.

19. James has $2\frac{1}{3}$ times as many dollars as John, and the difference between their sums is \$40; how many dollars has each?

20. Write down the cube of $m-n$.

21. Find the value of $\frac{2a}{3x^2}\times\frac{4x^3}{a^2}\times\frac{9a^3}{16x^2}-\frac{a^5x}{x^4}\div\frac{a^3}{x^2}$.

22. Factor $8x^2+7x-46$.

23. Divide the difference of the squares of $3x^2-4x+5$ and $3x^2+4x-5$ by the sum of the quantities.

24. Solve $\frac{x+4}{3}-9=\frac{x+9}{7}-\frac{4x+4}{3}$.

25. Factor $x\ +5xy+4y^2+5x+5y$.

26. Find the value of $(77x+19y)^3+(23x+81y)^3$ when $x=73$ and $y=-73$.

27. Multiply $6a-8b$ by $6a-7b$ and divide the product by $3a-4b$.

28. Factor $28x^2-109x+88$.

29. Solve $\frac{2x+4}{2}-5+\frac{3x+9}{5}=x+3$.

30. Divide 100 into two parts so that if one part be divided by 6 and the other part by 4 the sum of the quotients will be 20.

31. If $x=4a+b$, $y=5a-3b$ and $z=2b$, find the value of $2x+4y+5z$.

32. Write down the co-efficient of x^3 in $(x^2+3x-5)(x^2-5x-1)$.

33. If $a=1$, $b=3$, $c=5$ and $d=0$, find the value of $a^2+2b^2+3c^2+4d^2$.

34. Factor $a^2x^2-a^2y^2$.

35. Multiply $\frac{x^2+x-2}{x^2-2x-3}$ by $\frac{x^2-x-2}{x^2+2x-3}$.

36. Divide x^3-5x^2-x+14 by x^2-3x-7.

37. If from 3 times a certain number we subtract 8, half the remainder will be equal to the number itself diminished by 2; what is the number?

38. Find the sum of $\frac{2}{x^3+x^2+x+1}$ and $\frac{3}{x^3-x^2+x-1}$.

39. Simplify $\frac{a^3+3a^2b+3ab^2+b^3}{a^3-3a^2b+3ab^2-b^3}\times\frac{a^2-2ab+b^2}{a^2+2ab+b^2}$.

40. Divide $x^4+10x^3+35x^2+50x+24$ by $(x+1)(x+4)$.

41. Divide $\frac{a^2x+ax^2+x^3}{a^3-x^3}$ by $\frac{x}{a-x}$.

42. Divide $x^4+x^3-19x-4x^2-15$ by x^2-2x-3.

43. Simplify $\frac{6a^2-17a+12}{12a^2-25a+12}+\frac{27a^2+18a-24}{12a^2+7a-12}+\frac{25a^2-25a+6}{20a^2-23a+6}$.

44. James said to John if you give me $\frac{1}{6}$ of your money I will have \$170; then John said to James if you will give me $\frac{1}{3}$ of your money I will have \$170. What did each have?

45. Put $(x^2-5x+4)(x^2+5x+4)$ into 4 linear factors.

46. Simplify $\frac{3x^2+10xy+3y^2}{3x^2+8xy-3y^2}$.

47. Write down the product of $(x+y+z)(x^2+y^2+z^2-xy-xz-yz)$.

48. Find two equal factors of $4x^2-12xy+9y^2+4xz-6yz+z^2$.

49. Find the value of $\frac{a+b}{a-b}$ when $a=\frac{1}{2}$ and $b=\frac{2}{3}$.

50. Simplify $\frac{a^4-b^4}{a^2+2ab+b^2} \div \frac{a-b}{a^2+ab}$.

51. Find the value of $a^3-b^3+c^3+3abc$ when $a=\cdot 03$, $b=\cdot 1$ and $c=\cdot 07$.

52. Solve $\frac{17-3x}{5}=\frac{29-11x}{3}+\frac{28x+14}{21}$.

53. Multiply a^3+a^2+a+1 by a^5-a^4+a-1.

54. Solve $6(x-1)+8(x+2)=27(x-3)$.

55. a^2b+4 is a factor of $a^4b^2+a^2b-12$, find the other.

56. If $a=1$, $b=2$, $c=-3$, find value of $a^3+8b^3+c^3-6abc$.

57. Simplify $42\left\{\frac{4x-3y}{6}-\frac{3x-4y}{7}\right\}-56\left\{\frac{3x-2y}{7}-\frac{2x-3y}{8}\right\}$.

58. Solve $\frac{7x+1}{20}-\frac{17-2x}{12}=\frac{5x+1}{16}$.

59. The product of two factors is $(9x+5y)^2-(5x+9y)^2$ and one factor is $x-y$. Find the other.

60. Reduce to its lowest terms $\frac{x^4+2x^2+9}{x^4-4x^3+10x^2-12x+9}$.

61. If $x+9$ will divide $x^3+16x^2+ax+18$ without a remainder, find a.

62. Factor $24x^2-70xy-75y^2$.

63. Find the sum of the squares of $mx+qy$; $qx-my$; $my+qx$ and $qy-mx$, and express the result in factors.

64. What number must be added to x^2+9x+4 in order that it may be divisible by $x+6$?

65. Solve $\frac{x}{2}-\frac{2x-11}{3}-\frac{x+3}{4}=0$.

66. Simplify $\frac{2ax+3a^2}{4x^2-3ax}\times\frac{4ax^3-3a^2x^2}{a^2x^2-a^4}\times\frac{a^2-x^2}{2x^2+3ax}$.

67. Multiply a^6+a^5-a-1 by $1-a+a^2-a^3+a^4$.

68. Solve $\frac{5x-9}{13}-\frac{23-2x}{9}=3x-20$.

69. Solve $\frac{10x-7}{2x-1}-\frac{9x-1}{3x+1}=2$.

70. Find the L. C. M. of $x^2-9x+20$ and $x^2+6x-55$.

71. Divide \$560 between A and B so that for every dollar A gets B shall get \$2.50.

72. Factor $2x^2-21x+55$.

73. Solve $\frac{3x-1}{7}+\frac{5-x}{4}-\frac{2x-4}{12}=2-\frac{x+2}{28}$.

74. Factor $2kl-2mn-k^2-l^2+m^2+n^2$.

75. Factor $a(a-b)+m(a-b)+l(a-b)$.

B.

1. Find co-efficient of x in the product of $(x+8)\,(x+3)\,(x+2)$.

2. Simplify $\frac{x+1}{x^2+x+1}+\frac{x-1}{x^2-x+1}$.

3. Find product of $(x+a)\,(x+b)\,(x-c)$.

4. From result in number three write the product of $(x+8)$ $(x+9)\,(x-7)$.

5. Divide $a^{10}+a^5+1$ by a^2+a+1 and multiply the quotient by $a^4+a^3+a^2+a+1$.

6. Simplify $\frac{x-2y}{4}-\frac{x+2y}{5}+\frac{5y}{6}$.

7. Solve $\frac{x}{2}+\frac{11}{2}=\frac{6x+10}{4}+\frac{6x-16}{8}+\frac{4}{12}$.

8. A farmer sold 2 calves and 3 sheep for $50; and 3 calves and 1 sheep for $40. Find cost of each.

9. Multiply $\frac{(a-x)x}{a}$ by $\frac{ax}{a^2-x^2}$.

10. What value of x will make x^2+9 equal to 58?

11. Subtract $(b-a)\,(c-d)$ from $(a-b)\,(c-d)$ and find value of result when $a=2b$ and $d=2c$.

12. Solve $(x+5)^2-(4-x)^2=21x$.

13. Simplify $\frac{3}{1+y}+\frac{5}{1-y}-\frac{6}{1-y^2}$.

14. What number is it whose half, third and fourth parts taken together are equal to 78?

15. Find value of $x^4-x^3-4x^2-3x-5$ when $x=3$.

16. What does $x^4-4ax^3+6a^2x^2-4a^3x+a^4$ equal when $x=a$?

17. Simplify $(a+b+3c)^2+(b+3c)^2-2(b+3c)\,(a+b+3c)$.

18. Multiply $a-b+c$ by a quantity which will give $a^3-(b-c)^3$ as product.

19. From the quotient of $(x^5-y^5)\div(x-y)$ take the quotient of $(x^5+y^5)\div(x+y)$.

20. A person has two kinds of wine, one at 40c. per qt. and the other at 24c. per qt. How much of each must he take to form a gallon worth $1.12?

21. Find the value of $x^3+y^3+z^3-3xyz$ when $x=4a+5$, $y=4a-5$, $z=-8a$.

22. Simplify $\left(\frac{1}{a}+\frac{1}{b}\right)\left(\frac{a^2+b^2}{a^2-b^2}\right)\left(\frac{4ab}{a-b}\right)$.

23. Find H.C.F. of x^3+4x^2+4x+3 and $x^3+3x^2+4x+12$.

24. What value of x will make $(x+3)(x+4)$ greater than $(x+7)(x-6)$ by 102?

25. If $x=2$, $y=3$, and $z=5$, find the value of $\frac{y-z}{x}+\frac{z-x}{y}+\frac{4x-3y}{z}$.

26. Write down the cube of $(4y-3z)$.

27. Simplify $\frac{x^2-y^2}{xy}\times\frac{x^2}{x+y}\div\frac{x-y}{y^2}$.

28. If $a^2-bc=x$, $b^2-ac=y$, $c^2-ab=z$, prove that $ax+by+cz=(x+y+z)(a+b+c)$.

29. Simplify $\frac{7x-10}{5}-\frac{3x-7}{6}-\frac{27x-30}{30}$.

30. Solve $\frac{1}{x}+\frac{1}{2x}-\frac{1}{3x}=\frac{7}{3}$.

31. Simplify $\frac{1}{(2-m)(3-m)}-\frac{2}{(m-1)(m-3)}+\frac{1}{(m-1)(m-2)}$.

32. Reduce to lowest terms $\frac{x^3-3x^2+7x-21}{2x^4+19x^2+35}$.

33. Factor $m^2-n^2-p^2+q^2+2(mq+pn)$.

34. Divide $x^2+\frac{1}{x^2}+2$ by $x+\frac{1}{x}$.

35. Factor $(x+y)^2+(x+y)(a+b)+ab$.

36. If $a+1$ and $a+2b$ are factors of $6a^2b+3a^2+12ab^2+12ab+3a+12b^2+6b$, find the other factor.

37. Divide $x^6-20a^3x^3+343a^6$ by $x^2+ax+7a^2$ and then divide the quotient by $x^2+4ax+7a^2$.

38. If $x^2+47x-m$ is divisible by $x+17$, find m.

39. Find the value of x that will make $x^3+3ax^2+4a^2x-9a^3$ equal to the cube of $x+a$.

40. Take $\frac{1}{1+x+x^2}$ from $\frac{1}{1-x+x^2}$.

41. Reduce to its lowest terms $\frac{x^4-2x^3-25x^2+26x+120}{x^4-4x^3-19x^2+46x+120}$.

42. Divide $1+\frac{n-1}{n+1}$ by $1-\frac{n-1}{n+1}$.

43. Which of the factors x^2-2x+3 or $2x^2+x-4$ is likely to belong to $2x^4-3x^3-2x^2+10x-8$, and why? Find the other factor.

44. Divide $\frac{x^2+5x-14}{x^2+7x+12}$ by $\frac{x^2+3x-10}{x^2+7x+12}$.

45. When $x=5$ and $y=3$, show that $(2x+y)^2-(2x-y)^2$ is equal to $8xy$.

46. Prove that $(a-2)^3-2(a-2)^2+3(a-2)-4=a^3-8a^2+23a-26$.

47. Find value of $(x-y)^3+(x-9y)(x-y)$ when $x=5=y+1$.

48. Multiply $1+ab+a^2b^2+a^3b^3$ by $1-ab+a^4b^4-a^5b^5$.

49. Divide $x^4+x^2y^2+y^4$ by x^2+xy+y^2, and hence find factors of $x^8+x^4y^4+y^8$.

50. There is a number of two digits of which the unit digit is three times the other, and if 54 be added to the number the digits are reversed. Find it.

51. Multiply $\frac{1-x^2}{1+y}$, $\frac{1-y^2}{x+x^2}$ and $1+\frac{x}{1-x}$ together.

52. $a^4+a^2b^2+b^4$ is divisible by a^2+ab+b^2 without a remainder. Apply this to divide x^8+16x^4+256 by x^4+4x^2+16.

53. Write down the co-efficient of x^2 in $(x^2-4x+9)(x^2+3x-5)$.

54. Divide $\frac{a^2+ab+ac+bc}{a^2-ac+ad-cd}$ by $\frac{a+b}{a+d}$.

55. Simplify $\frac{x^3+a^3}{2ax-x^2}\times\frac{x^3}{x^2-ax+a^2}\div\frac{4a^2x^2+2ax^3}{4a^2-x^2}$.

56. Find value of $\frac{3}{1+x}+\frac{3}{1-x}$ when $x=\frac{1}{2}$.

57. Divide $a^2-4ab+4b^2$ by $\frac{a^2-2ab}{a+2b}$.

58. Write down the cube of $\frac{a}{b}+\frac{b}{a}$.

59. Find product of $\frac{2x}{a}-\frac{3y}{b}$ and $\frac{4x^2}{a^2}+\frac{6xy}{ab}+\frac{9y^2}{b^2}$.

60. Simplify $\frac{1}{1+3a+2a^2}+\frac{1}{1-a-2a^2}$.

61. Find L.C.M. of a^2-1; a^2+2a-3; a^3-7a^2+6a.

62. Simplify $\left(\frac{1}{a+b}+\frac{1}{a-b}\right)\div\frac{c^2}{a^2-b^2}$.

63. Find value of $ax+by$ when $x=\frac{cq-br}{aq-bp}$ and $y=\frac{ar-cp}{aq-bp}$.

64. Reduce to lowest terms $\frac{4+12x+9x^2}{2+13x+15x^2}$.

65. Simplify $\left(\frac{x-1}{x^2+y^2}-\frac{y^2x-x^3}{y^4-x^4}\right)(y^2+x^2)$.

66. Factor $x^3-8y^3+z^3+6xyz$.

67. 〃 $x^3+8y^3+z^3-6xyz$.

68. 〃 $8a^3+27b^3-c^3+18abc$.

69. 〃 $8a^3-27b^3-c^3-18abc$.

70. 〃 $x^3+8y^3+6xy-1$.

71. 〃 $x^3-8y^3-6xy-1$.

72. 〃 $x^4-3x^3-3x^2+7x+6$.

73. 〃 $x^4+8x^3-10x^2-104x+105$.

74. 〃 $x^4-14x^3+71x^2-154x+120$.

75. 〃 $x^4+4x^3-49x^2-76x+480$.

76. Prove the following identity by factoring: $(9x^2-4y^2)(81x^4+16y^4+36x^2y^2)=(27x^3+8y^3)(27x^3-8y^3)$.

77. Divide $a^4+b^4+2a^2b^2-c^4-d^4-2c^2d^2$ by $a^2+b^2+c^2+d^2$.

78. Show that $(a^2-bc)^3+(b^2-ac)^3+(c^2-ab)^3-3(a^2-bc)(b^2-ac)(c^2-ab)$ is an exact square.

79. Factor $x^4+10x^3+17x^2-40x-33$.

80. Show without simplifying that $x+1$ is a factor of $mn(x^5+1)+(n^2+m^2)(x^4+x)-(n^2+2mn)(x^3+x^2)$.

81. Simplify $(a+b)(a+2b)(a-2b)(a-b)$.

82. Simplify $\dfrac{\dfrac{m^2+n^2}{n}-m}{\dfrac{1}{n}-\dfrac{1}{m}}\times\dfrac{m^2-n^2}{m^3+n^3}$.

83. Factor $4a^4-12a^3x+13a^2x^2-6ax^3+x^4$.

84. Solve $\frac{x}{2}+\frac{x}{11}=\frac{3x+5}{2}+\frac{3x-8}{4}+\frac{1}{3}$.

85. Factor $x^2+6mn+5m^2-12mn-9n^2$.

86. Multiply $x^4-\frac{x^2}{y}+\frac{1}{y^2}$ by $x^4+\frac{x^2}{y}+\frac{1}{y^2}$.

87. Factor $(a+2b)^3+(2a+b)^3$.

88. Show that $(a-b)^2+\frac{6a^2b+2b^3}{a-b}=\frac{(a+b)^3}{a-b}$.

89. Find an expression that will exactly divide $x^3+2ax^2+a^2x+2a^3$ and $x^3-2ax^2+a^2x-2a^3$.

90. If $4x^4+28x^3y+Rx^2y^2+42xy^3+9y^4$ is a perfect square, find R.

EXERCISE I.

1. Solve $\frac{x}{3}+\frac{x}{4}+\frac{x}{2}=x+8$.

2. " $\frac{1}{2x}+\frac{2}{3x}-\frac{3}{4x}=\frac{5}{6}$.

3. " $\frac{2x+1}{2x-1}=\frac{5}{4}$.

4. " $\frac{128}{3x-4}=\frac{216}{5x-6}$.

5. Find the value of $(2x+y)\ (x+y)+(2y+z)\ (y+z)+(2z+x)\ (z+x)$ when $x=1$, $y=2$, $z=-3$.

6. Find, by Horner's method of division, the quotient of $21x^5-2x^4-70x^3-23x^2+33x+27$ divided by $7x^2+4x-9$.

7. Show how to find the product of $(m-n+p-q)\ (m-n-p+q)$ without multiplying out, and write it down.

8. Multiply a^2+ab+b^2 ; a^2-ab+b^2 ; and $a^4-a^2b^2+b^4$.

9. Factor $a^2-ab-6b^2$; $a^4+b^4-23a^2b^2$ and $15x^2+8xy-16y^2$.

10. Show without multiplying out that 7^4-1 is divisible by 6.

EXERCISE II.

1. Simplify $\frac{3(x^2+x-2)}{x^2-x-2}-\frac{3(x^2-x-2)}{x^2+x-2}-\frac{8x}{x^2-4}$.

2. Factor $1-a^2-b^2+c^2+2ab+2c$.

3. Divide $x^3+y^3+z^3-3xyz$ by $x+y+z$.

4. Show that $(a-b)^3+3ab(a-b)=(a+b)^3-3ab(a+b)-2b^3$ if $a=3$, $b=2$.

5. Simplify $\frac{(x^2-xy+y^2)\,(x^3-y^3)\,(x+y)^2}{(x^4+x^2y^2+y^4)\ (x^2-y^2)^2}$.

6. Express $(a^2-b^2)\ (c^2-d^2)$ as the difference of two squares.

7. Factor $6x^2-5xy-6y^2$; $x^{12}+y^{12}$ and $2a^2+5a+7ab+6b^2+8b+2$.

8. Simplify $\frac{a-3}{a-2}-\frac{2a+3}{a-1}-\frac{11-6a}{a^2-3a+2}$.

9. Simplify $\frac{x^2+\frac{5}{6}x-1}{x^2-\frac{13}{6}x+1}$.

10. Express $a^2(c-b)+b^2(a-c)+c^2(b-a)$ as the product of three binomial factors.

EXERCISE III.

1. Solve $3x-4\{9-(2x+7)\}+3x=13$.

2. Solve $\frac{24x+32}{8x+4}-\frac{6x+114}{3x+36}=1$.

3. Divide \$2280 among A, B, C, giving A $\frac{2}{5}$ more than B, and B \$240 less than C.

4. Divide the product of (x^2-xy+y^2) and (x^2+xy+y^2) by the quotient of x^3-y^3 by $x-y$.

5. Find H.C.F. of $6x^3+x^2y+6xy^2+35y^3$ and $14x^3-23x^2y+52xy^2-7y^3$.

6. Add 1 to the continued product of x, $x+1$, $x+2$ and $x+3$ and divide the result by x^2+3x+1.

7. Factor $4x^4+4x^2y^2+4y^4$; $b^2+c^2-a^2-2bc$, and $2a^2+3ax-2x^2$.

8. Solve $\frac{2(x+1)}{15}+\frac{x-1}{3}+5=x$.

9. Solve $\frac{3x-13}{8}-\frac{4x+6}{9}=1-\frac{x-1}{10}$.

10. Show by Horner's method of division that when x^7+1 is divided by x^2+x+1 the remainder is $x+1$.

EXERCISE IV.

1. Simplify $3(x+z)-(6y-z)-2\{x-(2y+z)-(y-3z)\}$.

2. Find by factoring the L.C.M. of x^2+5x+6 ; x^2+2x-3 ; and $x^4+x^3-2x^2$.

3. Distinguish between an identity and an equation. What value of C makes $(x-2)^2-(x-1)(x-3)=C$ an identity? Will any value which does not involve x make it an equation?

4. Show that the difference of the squares of any two consecutive numbers is equal to the sum of the numbers.

5. Multiply $x+1+\frac{1}{x}$ by $x-1+\frac{1}{x}$.

6. Simplify $\frac{\{(x+y)^2-xy\}\{(x-y)^2+xy\}}{x^4+x^2y^2+y^4}$.

7. Write down quotient of $(a^2+ab+b^2)^3+(a^2-ab+b^2)^3$ by a^2+b^2.

8. Solve $\frac{2x}{9}-2=\frac{3x-16}{18}$.

9. Solve $\frac{6x-3}{3x-8}-\frac{x-1}{x-4}-1=0$.

10. Divide $(x+y)^3-3(x+y)^2z+3(x+y)z^2-z^3$ by $x+y-z$.

EXERCISE V.

1. Show that $(mx+ny+pz)^3+(px-ny+mz)^3$ is divisible by $(m+p)(x+z)$.

2. From $(a-b)x-(b-c)y$ take $(a+b)x+(b+c)y$ and divide the difference by $x+y$.

3. What is meant by a co-efficient? Find without multiplying out, the co-efficient of x in the expansion of $(x+4)(x+5)(x+6)$.

4. Find the co-efficient of x when $x^3+4x^2-110x-63$ is divided by $x-9$.

5. Multiply $a^2+b^2-c^2+2ab$ by $c^2+a^2+b^2+2ab$.

6. Divide $x^3+3ax^2+3a^2x+a^3+b^3$ by $x+a+b$.

7. If $a=1$, $b=2$, $c=3$, $d=4$, find the value of

$$\frac{a+b}{a-b}+\frac{4b-c}{b+c}+\frac{c+d}{c-d}-\frac{7d-(a+b)}{c+d-b}.$$

8. Find by factoring the L.C.M. of x^2+x-2; x^2-4x+3 and x^2-x-6.

9. Simplify $(x+y+z)^2+(x+y-z)^2+(x-y+z)^2+(-x+y+z)^2$.

10. Solve $x-\frac{x-1}{2}-\frac{x}{3}=\frac{x-1}{3}-\frac{x-2}{4}+\frac{x-3}{5}$.

EXERCISE VI.

1. What is the value of a when $x^4+ax^3-6x^2+3x-21a$ vanishes if $x=4$?

2. The product of two algebraical expressions is x^7-64x and one is x^2-4, find the other.

3. Factor $x^6-3x^5+3x^4-x^3-8$.

4. Factor $36x^3+27x^2-20x-15$.

5. Write down the cube of $x+8y$.

6. Solve $\dfrac{x-\frac{1}{a}}{c}+\dfrac{x-\frac{1}{b}}{a}+\dfrac{x-\frac{1}{c}}{b}=0$.

7. Simplify $\frac{a+b}{ab}(a^2+b^2-c^2)+\frac{b+c}{bc}(b^2+c^2-a^2)+\frac{a+c}{ac}(a^2+c^2-b^2)$.

8. A train carries first-class passengers at 4c., second at 3c., and third at 2c. a mile. There are 12 times as many third-class as second-class; five times as many second as first. The whole fare was $11.12 per mile. How many are first-class?

9. Show that $(a+b-c)^2-(a-b+c)^2=4a(b-c)$.

10. Prove by division that $\dfrac{a^3-\frac{1}{a^3}}{a-\frac{1}{a}}=a^2+1+\frac{1}{a^2}$.

EXERCISE VII.

1. Find value of $\frac{x^2+y^2-z^2}{x-y+z}-\frac{x^2-y^2-z^2}{x+y+z}$ when $x=4$, $y=5$, $z=6$.

2. Employ factors to find the result of dividing $a^4+b^2c^2-a^2c^2-a^2b^2$ by $ac+a^2-bc-ab$.

3. Write down the quotient of $(4x+3y-2z)^2-(3x-2y+3z)^2$ divided by $x+5y-5z$.

4. Simplify $\frac{\{(a+b)^2+(a-b)^2\}\{(a+b)(a-b)\}}{a^3+a^2b+ab^2+b^3}$.

5. If $(x-5)^2-(x-7)\ (x-3)=a$, find the value of a in its simplest form.

6. Find the value of $\left(\frac{x^2}{y}-\frac{y^2}{x}\right)\left(\frac{x}{y^2}+\frac{y}{x^2}\right)\div\left(\frac{x^2}{y}+\frac{y^2}{x}\right)\left(\frac{x}{y^2}-\frac{y}{x^2}\right)$ when $x=4$, $y=3$.

7. Prove that x^2-3x-a; and x^2-4x-5 are both divisible by the same quantity if $a=10$, and find it.

8. Solve $\frac{x^2-5x+4}{x-4}+\frac{x^2-5x+6}{x-3}=3$.

9. The sum of two numbers is 35, the difference exceeds $\frac{1}{5}$ of the smaller by 2, find the numbers.

10. Solve $7(x-2)-5(2x-9)=\frac{1}{2}(x+13)$.

EXERCISE VIII.

1. Solve $(2x-3)^2-(2x-7)^2=5(x+3)$.

2. Factor $y^2z-yz^2+z^2x-zx^2$; $x^2-(a+\frac{1}{a})x+1$ and $x^2-3x(a+b)-4(a+b)^2$.

3. Find the value of $(y-z)^2+(z-x)^2+(x-y)^2$ when $x=-1$, $y=0$, $z=1$.

4. Find by factoring the L.C.M. of $5x^2-15x+10$, $6x^2-6x-12$ and $12x^2-12$.

5. Reduce to lowest terms $\frac{x^4+4x^3-19x^2-46x+120}{x^4-25x^2+144}$.

6. Simplify $\frac{2a}{(x-2a)^2}-\frac{x-a}{x^2-5ax+6a^2}+\frac{2}{x-3a}$.

7. Simplify $\frac{x(x+a)(x+2a)}{3a}-\frac{x(x+a)(2x+a)}{6a}$.

8. Determine a and b such that in the product of x^2+x+1 and x^3+ax^2+bx+c the co-efficients of x^4 and x^3 may vanish.

9. Show that $(5x-3y-4)^2-(3x+7y+4)^2$ is exactly divisible by $2x+y$.

10. Express $\frac{x+y}{xy}\left(\frac{1}{x}-\frac{1}{y}\right)-\frac{y+z}{yz}\left(\frac{1}{z}-\frac{1}{y}\right)$ as the difference of two squares.

EXERCISE IX.

1. Divide $x^5-\frac{1}{x^5}$ by $x-\frac{1}{x}$.

2. Find the product of $(a+b)(a^2+ab+b^2)(a-b)(a^2-ab+b^2)$.

3. Show that $(x-y)^3+(y-z)^3+(z-x)^3=3(x-y)(y-z)(z-x)$.

4. Solve $\frac{x-9}{x-5}+\frac{x-5}{x-8}=2$.

5. Divide $(x^2-yz)^3+8y^3z^3$ by x^2+yz.

6. At what times between 4 and 5 o'clock are the hands of a watch at right angles to each other?

7. Show that $3(a-b)(a-c)+2(b-c)(c-a)=3(a-b)^2$.

8. Solve $a(x-a)+b(x-b)+2ab=0$.

9. Use Horner's method to find the quotient of $x^{12}+x^6-2$ divided by x^4+x^2+1.

10. Find the continued product of $x+a$, $x+b$, $x+c$, and from the result write down the product of $a-m$; $a-n$; $a-p$.

EXERCISE X.

1. What must be added to $(a+b+c)^2$ that the sum may be $(a-b-c)^2$?

2. What quantity must be multiplied by $x+1$ to give x^3+3x^2+3x+1?

3. What must be multiplied by $x-\frac{1}{x}$ to give $x^3-\frac{1}{x^3}-\left(x-\frac{1}{x}\right)^2$?

4. The price of barley per bushel is 15 cents less than wheat, and the value of 50 bushels barley exceeds that of 30 bushels wheat by $5.50. Find price of each per bushel.

5. Examine whether x^2-5x+6 is a factor of $x^3-9x^2+26x-24$ and find the other factor.

6. Show that $a^5+a^2b^2-ab^2-b^3$ has a^2-b as a factor, and find the other.

7. Solve $\frac{x}{a}+b=\frac{x}{c}+d$.

8. Solve $\frac{x}{a}+\frac{x}{b}+\frac{x}{c}=1$.

9. Divide $m^2-\left(n+\frac{1}{n}\right)m+1$ by $m-n$.

10. Solve $\frac{8x+10}{20}-\frac{19x+6}{21x-2}=\frac{2x-3}{5}$.

EXERCISE XI.

1. Divide the product of a^2+ax+x^2 and a^3+x^3 by $a^4+a^2x^2+x^4$.

2. Show that $(1+x+x^2+\ldots\ldots x^{n-1})\,(1-x+x^2-x^3+\ldots\ldots x^{n-1})$ is equal to $1+x^2+x^4+\ldots\ldots x^{2n-2}$.

3. Find what value of x will make $5(x-3)-4(x+1)$ equal to 76.

4. Find by factoring the quotient of $a^3+a^2b+a^2c-abc-b^2c-bc^2$ by a^2-bc.

5. What must be the value of x, y, z that $a^3+a^2x+ay+z$ may have $a-1$, $a-2$, $a-3$ as factors?

6. Divide $x^3+(m+n+p)x^2+(mn+mp+np)x+mnp$ by $x+p$.

7. Find without actual multiplication the product of $\frac{x^2y^2}{9}-xy+9$ by $\frac{xy}{3}+3$.

8. Simplify $\frac{x^2-6x+8}{x^2-4x+3}\times\frac{x^2-5x+6}{x^2-2x-8}\div\frac{(x-2)^2}{x^2-1}$.

9. When $x=\frac{a+1}{ab+1}$ and $y=\frac{ab+a}{ab+1}$ reduce $\frac{x+y-1}{x+y+1}$ to its simplest form in terms of a and b.

10. Prove $\frac{x^3-y^3}{x-y}=(x+2y)^2-3y(x-y)$.

EXERCISE XII.

1. By what binomial must x^3-3x-2 be multiplied that it may be a perfect square?

2. Show that $x^4+y^4+(x+y)^4=2(x^2+xy+y^2)^2$.

3. Show that $1+\frac{a^2+b^2-c^2}{2ab}=\frac{(a+b+c)\,(a+b-c)}{2ab}$.

4. A cistern is $\frac{3}{4}$ full of water, but 220 gallons are run off, and it is then $\frac{1}{5}$ full. How many gallons will it hold?

5. Find the value in its simplest form of

$$\left(\frac{2a}{3x^2}\times\frac{8x^3}{3a^2}\times\frac{27a^3}{32x^2}\right)-\left(\frac{a^5}{a^3x^3}\div\frac{a^3}{a^3x^2}\right).$$

6. Find H.C.F. of $6x^3-19x^2-16x-3$ and $2x^3-11x^2+11x+6$.

7. Factor $x^2+4x+4-4a^2+4ay-y^2$.

8. If $x=-2$, find the value of y in the equation $7x+18y=4$.

9. If $\frac{3}{8}$ of my money is equal to $\frac{5}{16}$ of it and \$47, what is the sum?

10. Factor $x^4+6x^3+27x^2+162x+729$.

EXERCISE XIII.

1. Factor $16x^4 - 24x^3 - 16x^2 + 12x + 4$.

2. If a, b, c be three consecutive numbers, c being the greatest, show that the difference between the squares of a and b is 3 less than c.

3. Solve $\frac{6x+7}{3x+1} = \frac{2x+19}{x+7}$.

4. Factor $3x^3 - 14x^2 - 24x$.

5. The express train from Sarnia to London travels 32 miles per hour and reaches London in 2 hours less time than the mixed train at 16 miles per hour. Find the distance.

6. Multiply $x^8 + x^6 + x^4 + x^2 + 1$ by $x^2 - 1$.

7. Express in 4 factors, $3(6x^2 + 5x)^2 - 10(6x^2 + 5x) - 8$.

8. Divide $(a^2 - bc)^3 + 27b^3c^3$ by $a^2 + 2bc$.

9. Reduce $\frac{4x^3 - 27x^2 + 58x - 39}{x^4 - 9x^3 + 29x^2 - 39x + 18}$ to its lowest terms.

10. Simplify $\frac{a}{b} - \frac{(a^2 - b^2)x}{b^2} + \frac{a(a^2 - b^2)x^2}{b^2(b + ax)}$.

EXERCISE XIV.

1. Find H.C.F. of $18x^4 + 17x^3 - 128x^2 - 14x + 9$ and $24x^4 + 22x^3 - 171x^2 - 14x + 11$.

2. Find by factoring L.C.M. of $9a^2 - 36x^2$, $4a^2 - 4ax + x^2$, $2a^2 + 3ax - 2x^2$.

3. A herd of cattle cost $720, but two were stolen and the average cost per head was then $4 more than at first. Find the number.

4. Divide $x^3 - (a + b - c)x^2 + (ab - bc - ca)x + abc$ by $x + c$.

5. Factor $x^3 - y^3 - x(x^2 - y^2) + y(x - y)$.

6. Simplify $\frac{x^2 - 25}{x^2 - 49} \times \frac{x^2 - 4x - 21}{x^2 + 8x + 15} \div \frac{2x - 10}{x + 7}$.

7. Find by factoring the quotient of $a^3 - (2b - 3c)^3$ divided by $a - (2b - 3c)$.

8. Simplify $\frac{4 + 12x + 9x^2}{2 + 13x + 15x^2}$ and test result by putting $x = 1$.

9. Divide $1 + z^3 + y^3 - 3yz$ by $1 + y + z$.

10. Solve $\cdot 5x - \cdot 75 - 1\cdot 8x + 1\cdot 2 = 1\cdot 5x - 9\cdot 5$.

EXERCISE XV.

1. Find by factoring the value of $\frac{\frac{1}{9}-\frac{1}{4}}{\frac{1}{3}-\frac{1}{2}}$.

2. Factor $(x^2+a-1)^2-a^2x^2$.

3. Factor $2(a^3+a^2b+ab^2)-(a^3-b^3)$.

4. Show that $(x+y)^2+(y+z)^2+(z+x)^2+2(x+y)\ (x+z)+2(x+y)\ (y+z)+2(y+z)\ (x+z)=4(x+y+z)^2$.

5. Simplify $\frac{(y+z-2m)^2-(z+m-2y)^2}{(m+y+z)^2-(m+y-5z)^2}$.

6. Show that $\frac{(ac+bd)^2}{(a^2+b^2)\ (c^2+d^2)}$ is always a proper fraction.

7. Divide y^6-2y^3+1 by y^2-2y+1.

8. Factor $x^2+x(m+n+p)+p(m+n)$.

9. Find three numbers whose sum is 51, and of which the greatest exceeds the least by 6, and the third is one-half of the other two.

10. Show that $b+d$ is a factor of $(a+b+c+d)^2-(a-b+c-d)^2$.

EXERCISE XVI.

1. Show that $x(y+z)^2+y(x+z)^2+z(x+y)^2-4xyz=(y+z)\ (z+x)\ (x+y)$.

2. Solve $\frac{12}{x}+\frac{1}{12x}=\frac{29}{24}$.

3. A man bought tea at 78c. per lb. and $\frac{1}{3}$ as many again lbs. of coffee at 30c. ; he sold tea at 96c. and coffee at 27c., and gained \$12.60. How many lbs. of each did he buy ?

4. Show that $\left(a+\frac{1}{a}\right)^2-\left(b+\frac{1}{b}\right)^2=\left(ab-\frac{1}{ab}\right)\left(\frac{a}{b}-\frac{b}{a}\right)$.

5. Simplify $\frac{a^3+(1+a)ab+b^2}{a^4-b^2}$.

6. If $a+b+c=0$ show that $a^3+b^3+c^3=3abc$.

7. Factor $x^2y^2-x^2-y^2+1$; $a^2(b-c)+b^2(c-a)+c^2(a-b)$.

8. Solve $\frac{4}{x+2}+\frac{7}{x+3}=\frac{37}{x^2+5x+6}$.

9. Reduce to its lowest terms $\frac{a^2+a+b^2+b+c^2+c+2ab+2ac+2bc}{a^2-b^2-c^2-2bc.}$.

10. Simplify $\frac{(a+b)\ (1-ab)}{(1-ab)^2-(a+b)^2}-\frac{a(1-b^2)+b(1-a^2)}{(1-a^2)(1-b^2)-4ab}$.

EXERCISE XVII.

1. Solve $\frac{x+4}{8}+\frac{x+5}{15}=\frac{x+2}{6}+\frac{6x+3}{45}$.

2. If $a^2+b^2=1=c^2+d^2$ show that $(ac-bd)^2+(ad+bc)^2=1$.

3. Simplify $\frac{b-c}{a^2-(b-c)^2}+\frac{c-a}{b^2-(c-a)^2}+\frac{a-b}{c^2-(a-b)^2}$.

4. Divide $(x^2-2yz)^3-27y^3z^3$ by x^2-5yz.

5. Find the value of a and b when $x^2-ax+12$ will divide $x^3-bx^2+75x-108$ without a remainder.

6. Simplify $(x+1)(x+2)(x+3)-(x-1)(x-2)(x-3)$.

7. Show that $2(a^2+b^2+c^2-ab-ac-bc)=(b-c)^2+(c-a)^2+(a-b)^2$.

8. Find two consecutive numbers such that the half and fifth part of the less may be equal to the third and fourth parts of the greater.

9. Show that $(b-c)^3+(c-a)^3+(a-b)^3-3(b-c)(c-a)(a-b)=0$.

10. Show that $a^3+b^3+c^3-3abc-a(a^2-bc)-b(b^2-ca)-c(c^2-ab)=0$.

EXERCISE XVIII.

1. If $x=b+c-2a$, $y=c+a-2b$, $z=a+b-2c$ prove $x^2+y^2+z^2+2xy+2xz+2yz=0$.

2. Simplify $\frac{b^2-c^2}{b+c}+\frac{c^2-a^2}{c+a}+\frac{a^2-b^2}{a+b}$.

3. " $\frac{1}{1-x}-\frac{1}{1+x}-\frac{2x}{1+x^2}-\frac{4x^3}{1+x^4}-\frac{8x^7}{1+x^8}$.

4. " $\frac{x-2a}{x+a}+\frac{x+a}{x-a}-\frac{x^2+3a^2}{x^2-a^2}$.

5. Prove that $(x+y)(x-y)+(y+z)(y-z)+(z+x)(z-x)=0$.

6. Prove that $a(b-c)+b(c-a)+c(a-b)=0$.

7. Show that $(a+b)^3+3c(a+b)^2+3c^2(a+b)+c^3=(b+c)^3+3a(b+c)^2+3a^2(b+c)+a^3$.

8. Simplify $(a+b+c)^2-(a+b)^2-(b+c)^2-(c+a)^2+a^2+b^2+c^2$.

9. Divide 1760 yds. into two parts such that half of one part with 200 yards may be double the other part.

10. If $p=a+b+c$, $q=ab+ac+bc$, $r=abc$, prove $pq-r=(b+c)(c+a)(a+b)$.

EXERCISE XIX.

1. Simplify $(3x-2)^2+(3x-2)(3x-4)+(3x-2)(3x-5)$.

2. If $x^2-yz=a$, $y^2-xz=b$, $z^2-xy=c$, show that $x^3+y^3+z^3-3xyz=ax+by+cz$.

3. Required the number whose $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ parts together are as much greater than 223 as $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$ of it are less than the same.

4. Add together the squares of $ax+by$ and $ay-bx$ and subtract the sum from the product of (a^2+y^2) and (b^2+x^2).

5. Factor $\frac{64a^3}{125}-\frac{27b^3}{8}$.

6. Simplify $\{(7x^2+4x-3)(7x^2-4x-3)\} \div \{(7x-3)(7x+3)\}$.

7. Solve $\frac{x-1}{34}+\frac{x-2}{51}-\frac{x-3}{102}=0$.

8. Solve $x-\frac{x-2}{3}=\frac{x+15}{4}-\frac{x}{5}$.

9. Show that $(a^2-bc)+(b^2-ca)+(c^2-ab)$ is not changed if a, b, c is each increased or diminished by the same quantity.

10. Divide $32x^5+243$ by $2x+3$ by factoring.

EXERCISE XX.

1. How many lbs. tea at 36c. and 60c. must be mixed to make 200 lbs. worth $86.40 ?

2. Write down the product of $(14x-17)(14x+17)$.

3. If $(x+1)^2=x$, find value of $11x^3+8x^2+8x-2$.

4. Factor $36x^4-97x^2y^2+36y^4$.

5. The perimeter of a square field is 588 yds. and of another 672 yds. Find the perimeter of another equal in area to both.

6. What must be added to $6x^4-x^3+x^2-x+1$ to make it an even multiple of x^2-x+1 and $x-1$?

7. What value of a will make $3x^4-7x^3+2ax^2-11x+a$ exactly divisible by x^2-x+1 ?

8. Find the value of $\frac{x-x^5}{1-x}$ when $x=1$.

9. Factor $6a^2-7ab+2ac-20b^2+64bc-48c^2$.

10. If $a^2+ab+529$ is a perfect square, find b.

EXERCISE XXI.

1. If a, b, c, d, e, represent the first 5 numbers and $x=0$, find the numerical value of $x^2(a+b+c+d+e)-a^2(b-c)-b^2(c-d)-c^2(d-e)-d^2(e-x)$.

2. If $x+y=a$, $y+z=b$, $z+x=c$, $a^2+b^2+c^2=0$, show that $xy+xz+yz=\frac{1}{2}(ab+ac+bc)$.

3. Find H.C.F. of $a^5+6a^4+11a^3+5a^2-3a-2$ and $a^6+3a^5-6a^3-5a^2-3a-2$.

4. If $x-2y$ is one factor of $x^3-2x^2y-4xy^2+8y^3$, find the others.

5. If $4x^4+12x^3y+ax^2y^2+6xy^3+y^4$ is the square of $2x^2+3xy+ky^2$, find a and k.

6. What three linear expressions divided into $x^3-7x+10$ will each give a remainder of 4?

7. If $a+\frac{1}{b}=1$ and $c+\frac{1}{a}=1$, show that $b+\frac{1}{c}=1$.

8. If $x=19$, find value of $\frac{7x+5}{23}+\frac{9x-1}{10}-\frac{x-9}{5}+\frac{2x-3}{15}$.

9. Find the equal factors of $9x^4-6x^3+43x^2-14x+49$.

10. Show that the sum of every fraction and its reciprocal is equal to or greater than two, and that $\frac{a}{b}+\frac{b}{c}+\frac{c}{d}+\frac{b}{a}+\frac{c}{b}+\frac{d}{c}>6$.

EXERCISE XXII.

1. Is $(a^2+c^2)(b^2+d^2)$ greater or less than $(ab+cd)^2$ when $ad=bc$?

2. If $m=2x^2-16x+14$, $n=x^2-5x-14$, factor the sum of $m+n$.

3. Show that $(5x^2+4xy+y^2)^2-(3x^2+4xy+y^2)^2$ is equal to $4x^2(2x+y)^2$.

4. Divide $\left(\frac{x^2}{a^2}+\frac{a^2}{x^2}-2\right)^2$ by $\frac{a}{x}-\frac{x}{a}$.

5. Find the value of x that will make $x^3+6x^2c+8xc^2+10c^3$ equal to the cube of $x+2c$.

6. Factor $56x^2-129x-9797$; $27x^2+192x-4067$.

7. Find the remainder when $5x^3-8x^2+8x+7$ is divided by $5x-3$.

8. Show that the difference between $\frac{m}{m-q}+\frac{m}{m-r}+\frac{m}{m-s}$ and $\frac{q}{m-q}+\frac{r}{m-r}+\frac{s}{m-s}$ is the same whatever m may be.

9. A, B, C give \$400 to a society. B gives twice as much as A and \$20 more, C as much as A and B together. What did each give?

10. If $a=4$, $b=5$, by what must ax^2+bx+1 be multiplied to give $8x^3+(5a+2)x^2+(4b-3)x+3$.

EXERCISE XXIII.

1. If $ay+bx=a$, $by-ax=b$, then $x^2+y^2=1$.

2. Find the value of $a^3-b^3-(a-b)^2$ when $a+2b=13$; $2a+b=32$.

3. Solve $\frac{a}{x-a}-\frac{a}{7(x-a)}=\frac{2a}{x+7a}$.

4. Solve $\frac{ad-bc}{d(c+dx)}+\frac{b}{d}=\frac{2a-bx}{c+dx}$.

5. A tradesman after spending \$100 a year, increases the remainder of his property by $\frac{1}{3}$ of itself, and at the end of 3 years his original capital is doubled. What had he at first?

6. Show that $x^2+y^2+z^2-xy-xz-yz$ is not changed by adding the same quantity to x, y, z.

7. Factor $x^3+8x^2-79x+70$.

8. Of the fractions $\frac{a-x}{a+x}$ and $\frac{a^2-x^2}{a^2+x^2}$, which is greater, when a is greater than x.

9. If $x+y+z=0$, prove that $x(x^2-yz)+y(y^2-xz)+z(z^2-xy)=0$.

10. If $x^2+u^2=2(xy+yz+uz-y^2-z^2)$, prove $x=y=z=u$.

EXERCISE XXIV.

1. In the expression x^3-2x^2+3x-4, substitute $a-2$ for x and arrange in descending powers of a.

2. If $x-y=1$, then $(x^2-y^2)^3=x^3-y^3+xy$.

3. Prove that $2(x^3+y^3+z^3-3xyz)\div x+y+z=(y-z)^2+(z-x)^2+(x-y)^2$.

4. If $x-y=2a$, show that $x^2-6ax+9a^2=(y-a)^2$.

5. A person leaves A for B at $3\frac{1}{2}$ miles per hr.; 40 minutes later another leaves B for A at $4\frac{1}{2}$ miles per hr., he goes $\frac{1}{2}$ mile more than half way when he meets the first traveller. How far from A to B?

6. Simplify $\frac{x^4-5x^2+4}{x^2+1} \times \frac{\frac{1}{x}-\frac{1}{x+2}}{x-\frac{1}{x}} \div \frac{2-\frac{4}{x}}{1+\frac{1}{x^2}}$.

7. Prove that $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.

8. Show without division that $x-a$ is a factor of $x^3-(a-m)x^2+(f-am)x-af$.

9. If $x-a$ and $x-b$ are each factors of x^2+x+1, then $a^3+b^3=2$.

10. If $a+b+c=0$, show that $a^2-b^2=bc-ac$.

EXERCISE XXV.

1. If $x+a$ and $x+b$ are each factors of x^3+mx^2+b then
$$m=\frac{a^2+ab+b^2}{a+b}.$$

2. Find the value of x which will make x^2-2x+3 a factor of $x^3-x^2+5x-21$.

3. Simplify $\left(\frac{a}{b}-\frac{b}{a}+\frac{b}{c}-\frac{c}{b}+\frac{c}{a}-\frac{a}{c}\right) \div (a-b)(b-c)(c-a)$.

4. Solve $\frac{2x-7a}{x-4a}+\frac{x-8a}{x-9a}=\frac{x-7a}{x-8a}+\frac{2x-9a}{x-5a}$.

5. Find the value of a for which the following fraction admits of reduction: $\frac{x^3-ax^2+19x-a-4}{x^3-(a+1)x^2+23x-a-7}$.

6. Simplify $\frac{\frac{3}{x}+1+\frac{x}{3}}{\frac{9}{x^2}-\frac{x}{3}}$.

7. Find by factoring what algebraical expression multiplied by itself gives $25x^4-20x^3-6x^2+4x+1$.

8. Solve $\frac{4x-11}{15}-\frac{2x-7}{6}+\frac{11}{90}=\frac{5x-14}{18}-\frac{3x-11}{9}$.

9. Prove algebraically that the sum of the squares of any four consecutive odd (or even) integers diminished by 20 is a square integer.

10. Show that x^7+1 divided by $(x+1)^2$ gives remainder $7(x+1)$.

EXERCISE XXVI.

1. Find that value of x that will make $x^3+3ax^2+4a^2x-9a^3$ equal to $(x+a)^3$.

2. If two numbers differ by d, show that the difference of their squares is d times their sum.

3. If $\frac{b^2+c^2-a^2}{2bc}=m$, and $\frac{(a-b+c)\ (a+b-c)}{(a+b+c)\ (b+c-a)}=n$, prove $(m+1)(n+1)=2$.

4. Find the value $(x-y)\ (x+y)-(x+y)^2$ when $3x+2y=45$ and $3y+2x=15$.

5. The L.C.M. of two quantities is $a^4-5a^2b^2+4b^4$ and the H.C.F. is a^2-b^2, one quantity is $a^3-2a^2b-ab^2+2b^3$, find the other.

6. The H.C.F. is $m-7$, the L.C.M. $m^3-10m^2+11m+70$, one expression is $m^2-5m-14$, find the other.

7. Factor $x^2+25x+289$.

8. If $x^2-3x+2=0$, show that $x^4-10x^3+35x^2-50x+24=0$.

9. Divide $\frac{a-1}{a}+\frac{b-1}{b}+\frac{c-1}{c}-1$ by $2-\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$.

10. Show that $(x^2+6xy+4y^2)^5+(x^2+2xy+4y^2)^5$ is divisible by $(x+2y)^2$.

EXERCISE XXVII.

1. Show that $x^4+y^4+z^4-2x^2y^2-2x^2z^2-2y^2z^2$ is divisible by each of the four expressions $x \pm y \pm z$.

2. Simplify $\frac{56x^3-28x^2-42x+14}{42x^2-28x-14.}$

3. Show that the value of the difference between $\left(x+\frac{1}{x}\right)^2$ and $\left(x-\frac{1}{x}\right)^2$ is independent of x.

4. Show that any four consecutive odd (or even) numbers plus 16 may each be put in the form of two equal factors.

5. From $a+b-c$ take $\frac{1}{2}a-\frac{1}{2}b-\frac{1}{2}c$.

6. Show that $\frac{x+1}{\frac{1}{x}+1}=x$ when x is any number.

7. Find the value of $\frac{a^2+b^2}{a^3-b^3}$ when $a=3$, $b=2$.

8. Find co-efficient of x^5 in $(1+x+x^2+x^3, \text{etc.})^2$.

9. Of two square fields one exceeds the other by 100 acres and its side is 400 yds. longer. Find the length of the side of the smaller field.

10. Solve $a(x-a)+b(x-b)+c(x-c)=2ab+2ac+2bc$.

EXERCISE XXVIII.

1. Find the value of c if $x^4+2x^3-10x^2+3cx+2c$ is exactly divisible by x^2-3x+8.

2. From $16\left(\frac{3x-2y}{8}+\frac{5x-3y}{4}\right)$ take $32\left(\frac{9x-2y}{16}-\frac{3x-5y}{8}\right)$.

3. Factor $x^2-\left(\frac{a}{b}-\frac{b}{a}\right)x-1$.

4. Find the co-efficient of x^4 in the product of
$$1+\frac{x}{2}+\frac{x^2}{4}+\frac{x^3}{8}+\frac{x^4}{16} \text{ by } 1+\frac{x}{2}+\frac{x^2}{4}+\frac{x^3}{8}+\frac{x^4}{16}.$$

5. Write down the cube of $1+x+x^2$.

6. Write down the square of $1+\frac{x}{2}+\frac{x^2}{3}$.

7. The depth of a cistern at one end is twice that at the other; water to the depth of 18 inches is frozen and the water below at deep end is three times that at the other. What was the original depth at the deeper end?

8. Multiply $1+\frac{1}{2}a+\frac{1}{3}b$ by $1-\frac{1}{2}a+\frac{1}{3}b$.

9. Prove $\frac{(2y-x)^3-(2x-y)^3}{3(y-x)}+\frac{(2y-x)^3+(2x-y)^3}{y+x}=10x^2-16xy+10y^2$.

10. Simplify $a+b-(2a-3b)-(5a+7b)-(2b-13a)$.

EXERCISE XXIX.

1. Solve $\frac{x-a}{bc}+\frac{x-b}{ac}+\frac{x-c}{ab}=\frac{2}{a}+\frac{2}{b}+\frac{2}{c}$.

2. Simplify $\frac{a^5+b^5}{a+b}+\frac{a^5-b^5}{a-b}$.

3. What is the difference between $3a+m$ and $7am$ when $a=5$ and $m=9$?

4. Simplify $\frac{x^{a+b}}{x^{2c}}\times\frac{x^{a+c}}{x^{2b}}\times\frac{x^{b+c}}{x^{2a}}$.

5. Multiply $1-\frac{1}{2}x+\frac{1}{3}x^2$ by $1+\frac{1}{3}x-\frac{1}{2}x^2$.

6. If \$1600 be put out, part at 4% and part at 5%, and if the yearly income is \$73, how much is out at 4%?

7. Write down the cube of $\frac{a}{b}-\frac{b}{c}$.

8. Show that the product of any four consecutive numbers increased by 1 is a perfect square.

9. Prove that $(ax+by+cz)^2-(bx+cy+az)^2$ is divisible by $(a+b)x+(b+c)y+(c+a)z$, and also by $a(x-z)+b(y-x)+c(z-y)$.

10. Simplify $\frac{1}{a-b}+\frac{1}{b-c}+\frac{1}{c-a}+\frac{(a-b)^2+(b-c)^2+(c-a)^2}{2(a-b)(b-c)(c-a)}$.

EXERCISE XXX.

1. Simplify $\frac{(3y-2x)^3-(3x-2y)^3}{5(y-x)}+\frac{(3y-2x)^3+(3x-2y)^3}{y+x}$.

2. Prove $(a^2+b^2+c^2)\ (x^2+y^2+z^2)=(ay-bx)^2+(cx-az)^2+(bz-cy)^2+(ax+by+cz)^2$.

3. Add $am-cl-bn$ and $cn-al-bm$, and from the sum take $-cm-bl-an$, and divide the result by $b-c-a$.

4. Prove that if one quantity measures two others it will also measure the difference of any multiples of these two quantities.

5. Factor $(x+y+z+a)^2-(x-y-z+a)^2$.

6. Factor $20x^3+12ax^2+25bx^2+15abx$.

7. Find remainder without actual division of $(x^4-3x^3+4x^2-2x+1)\div(x^2-x+1)$.

8. Extract square root of $9-24x+58x^2-116x^3+129x^4-140x^5+100x^6$.

9. Simplify $\frac{a(s-a)^2+b(s-b)^2+c(s-c)^2-abc}{(s-a)(s-b)(s-c)}$ when $2s=a+b+c$.

10. Factor $7x^2-6y^2-xy+19x+33y-36$.

EXERCISE XXXI.

1. Apply Horner's method to find value of $5x^5+497x^4+200x^3+196x^2-218x-2000$, when $x=-99$.

2. Show without expansion that $(1+x+x^2)^3-(1-x+x^2)^3-6x(x^4+x^2+1)-8x^3=0$.

3. Show that $a^2(b+c)-b^2(c+a)+c^2(a+b)+abc$ is divisible by $a-b+c$.

4. What value of a and b will make $x^2+2ax+b^2$ the square of $x+5$?

5. A man is three times as old as his son, but 10 years ago he was 5 times as old. Find son's age.

6. Express $x^2+20y^2+70z^2+52yz+4xy+6xz$ as the sum of three squares.

7. What quantity will divide without remainder into $x^4-2ax^3+(a^2-b^2)x^2+2ab^2x-a^2b^2$ and $x^4-(a^2+b^2)x^2+a^2b^2$?

8. Factor $x^4+12x^3+50x^2+84x+33$.

9. Express x^4-28x^2+16 as the product of two quadratic factors.

10. Prove that $(3x^2-5x+8)^2-(2x^2-x+4)^2$ is divisible by $(x-2)^2$.

EXERCISE XXXII.

1. How much tea at 50c. per lb. must be mixed with 100 lbs. at $87\frac{1}{2}$c. per lb. that the mixture may be worth $62\frac{1}{2}$c. per lb.?

2. Show that $(9x-4y)^2-(7x+3y)^2$ is a multiple of $2x-7y$ and $16x-y$.

3. Extract sq. root of $9x^4-42x^3+115x^2-154x+121$.

4. Find the conditions that $mx^3+nx^2+32x+15$ may be divisible by $2x-3$ and $3x+1$ for all values of x.

5. Find the value of $(m-n)\ (m+n)-(m+n)\ (m+n)$ when $3m+2n=45$ and $3n+2m=15$.

6. Factor $112a^2+138ab-135b^2$.

7. Show that $(5x^2-3x+2)^2-(2x^2+3x-1)^2=3(7x^2+1)\ (x-1)^2$.

8. Simplify $\dfrac{xy(x-y)+yz(y-z)+xz(z-x)}{x^2+yz-zx-xy} \div \dfrac{z^2-yz-zx+xy}{z^2-x^2}$.

9. Find the conditions that $x^4-px^3+qx^2-rx+s$ may be exactly divisible by $x-a$.

10. Divide $\dfrac{2+m}{2-m}-\dfrac{2-m}{2+m}$ by $\dfrac{2+m}{2-m}+\dfrac{2-m}{2+m}$.

EXERCISE XXXIII.

1. Determine the value of p and q when the expression $4y^4-12y^3+py^2+qy+16$ is a complete square.

2. Show that $\dfrac{(2b-c-a)^3-(2c-a-b)^3}{(c-a)^3-(a-b)^3}=\dfrac{9(b-c)}{b+c-2a}$.

3. The product of any three consecutive integers being found, and also that of any other three, the difference of the products will be a multiple of the difference of the middle integers of the two sets.

4. Show that ax^3+bx^2+cx+d is divisible by x^2+b^2 if $ad=bc$.

5. If $x+a$ be the H.C.F. of x^2+px+q and x^2+mx+n, then

$$a=\frac{q-n}{p-m}.$$

6. If $x^3+2ax-3b^2$ is divisible by $x-a$, prove $a=b$ or $-b$.

7. What value of p will make $3x-4$ a measure of $18x^2-px+28$?

8. Resolve into 5 factors $x^{16}-65536$.

9. Factor $9x^3+48x^2+52x+16$.

10. Factor $2x^2+11xy+12y^2+7xz+13yz+3z^2$.

EXERCISE XXXIV.

1. If $x+y+z=0$, show that $\frac{x(y^3-z^3)}{y-z}+\frac{y(z^3-x^3)}{z-x}+\frac{z(x^3-y^3)}{x-y}=0$.

2. Arrange $(x+y+z)a_1+(x+y-z)a_2+(x+z-y)a_3+(y+z-x)a_4$ in three terms involving x, y, z respectively with co-efficients a_1, a_2, a_3, a_4.

3. Find the condition that $x^3-3b^2x+2c^3$ may be divisible by $x-a$ whatever be the value of x.

4. $x^4-4x^3+6x^2-4x+1$ is a multiple of x^2-ax+1, find a.

5. Prove that the product of H.C.F. and L.C.M. of any two quantities is equal to the product of the quantities.

6. Simplify without expanding $(x^2+xy+y^2)^3+(x^2-xy+y^2)^3+6(x^2+y^2)(x^4+x^2y^2+y^4)$.

7. If $9x^4-30x^3y+ax^2y^2-10xy^3+y^4$ is a perfect square, find a.

8. Find value of $3x^5+54x^4+50x^3-19x^2-35x-18$ when $x=-17$.

9. Solve $\frac{10x+17}{18}-\frac{12x+2}{13x-16}=\frac{5x-4}{9}$.

10. If $bz-cy=p$, $cx-az=q$, $ay-bx=r$, then $ap+bq+cr=0$.

EXERCISE XXXV.

1. Find value of $2x^4-510x^3-513x^2+256x-1024$ when $x=256$.

2. Prove by factors that $(7x-3)^2+(7x-3)(x+2)+7x(x-4)-3(x-4)=(7x-3)(9x-5)$.

3. If $x=a+2b+3c$, $y=b+2c+3a$, $z=c+2a+3b$, show $x+y+z=6(a+b+c)$.

4. Factor $2y^2-5xy+2x^2-ay-ax-a^2$.

5. Reduce to lowest terms $\frac{3x^2-16x-12}{2x^3-16x^2-24x+288}$.

6. What number added to $\frac{x^2}{a^2}+\frac{a^2}{x^2}+\frac{m^2}{n^2}+\frac{n^2}{m^2}$ will make it a perfect square?

7. If $x^4+px^3+qx^2+rx+s$ be a perfect square, show that $r^2=p^2s$.

8. Find two equal factors of

$$\frac{x^4}{a^4}+\frac{a^4}{x^4}+\frac{x^2}{a^2}+\frac{a^2}{x^2}+2\left(\frac{x^3}{a^3}+\frac{a^3}{x^3}\right)+2\left(\frac{x}{a}+\frac{a}{x}\right)+4.$$

9. Show that the sum of the cubes of any three consecutive numbers is divisible by 3 times the second of them.

10. Simplify $\dfrac{\dfrac{x+y}{x-y}+\dfrac{x-y}{x+y}}{\dfrac{x+y}{x-y}-\dfrac{x-y}{x+y}}$.

EXERCISE XXXVI.

1. Solve $\frac{x}{2}+\frac{x}{3}-\frac{x}{4}+\frac{x}{5}=7\frac{5}{6}$.

2. Write down the square of $\dfrac{a^2+b^2-c^2-2ab}{a-b-c}$ in its simplest form.

3. Find a quantity which when multiplied into x^3-4x^2+5x-2 will make it a perfect square.

4. Find the simplest form of $\dfrac{(a+b)^n(a-b)^{n-1}}{(a+b)^{n-1}(a-b)^n}$.

5. By what quantities must x^3-x^2-x+1 be multiplied to make it a complete cube.

6. Find the values of $\left(\frac{x}{y}+\frac{y}{x}\right)$ and $\dfrac{x^3-y^3}{x-y}$ when $x=6$, $y=4$; will the values be the same if $y=6$ and $x=4$?

7. Find the quotient of $\frac{a^3}{8}-\frac{b^3}{27}$ by $\frac{a^2}{4}+\frac{ab}{3}+\frac{b^2}{9}$.

8. Prove that the difference of the squares of any two consecutive even (or odd) numbers is 4 times the intermediate number.

9. Find value of $x^3-8y^3+29z^3+18xyz$ when $2y=x+3z$ and $z=5$.

10. Multiply $(a+b)^2+(a-b)^2$ by $(a+b)(a-b)$, and divide result by $a^3+a^2b+ab^2+b^3$.

EXERCISE XXXVII.

1. If $a-b=1$, show that $(a-b)^2(a+b)^2=a^3-b^3+ab$.

2. If $a^2+a+1=0$, show that $a^3=1$.

3. What value of m will make $6x^4-2x^3+2mx^2+2x+m$ exactly divisible by x^2-x+1?

4. If $x+y-z=0$, show that $x^2+yz=y^2+xz$.

5. If $\frac{1}{a}+\frac{1}{b}=\frac{1}{a+b}$, show that $\frac{a}{b}+\frac{b}{a}+1=0$.

6. Prove that $x+y$ is divisible by $x^{\frac{1}{3}}+y^{\frac{1}{3}}$ and write the result.

7. If x is positive, show that $\frac{x}{x+1}+\frac{x+1}{x}-2$ is positive.

8. Which is greater $\frac{x}{y^2}+\frac{y}{x^2}$ or $\frac{1}{x}+\frac{1}{y}$, when x and y are positive?

9. Find two linear and one quadratic factor of $(x^2-yz)^2-(y^2-xz)(z^2-xy)$.

10. A courier passing through a certain place (P) goes $2\frac{1}{2}$ miles per hour; 4 hours later another passes through, going $3\frac{1}{2}$ miles per hour. How far will the second travel from (P) before he overtakes the first?

EXERCISE XXXVIII.

1. If $ac=b^2$, show that $(a+2b+c)=\frac{(b+c)^2}{c}$.

2. Show that $a-b$, $b-c$, $c-a$ cannot be all positive or all negative.

3. Show that a^3+b^3 is greater than a^2b+ab^2 unless $a=b$.

4. Prove $a+b=\frac{a^2}{a-b}+\frac{b^2}{b-a}$.

5. Find by factoring for what values of x are $\frac{x}{a}+\frac{a}{x}$ and $\frac{c}{a}+\frac{a}{c}$ equal to each other.

6. If $2a=y+z$, $2b=z+x$, $2c=x+y$, find the value of $a^4+b^4+c^4-2a^2b^2-2a^2c^2-2b^2c^2$ in terms of x, y, z.

7. If $ax^2+b^2y-c^3=0$, $ay^2+b^2x-c^3=0$, and $x+y-c=0$, then $b^2=ac$.

8. If $x^2=x-2$, show that $x^5=-x+6$.

9. If $x^2=x+1$, show that $x^5=5x+3$.

10. A sum of money is divided among A, B, C; A is to have \$120 less than half, B \$40 less than $\frac{1}{3}$ of it, C \$32 more than $\frac{1}{4}$ of it, what did each receive?

EXERCISE XXXIX.

1. Find by factoring the value of a in order that $x=2$ may be a solution of $\frac{ax}{a-1}+\frac{x}{2a}=\frac{13ax+2}{3ax}$.

2. If $\frac{1}{x-y}+\frac{1}{y-z}+\frac{1}{z-x}=0$, show that $x=y=z$.

3. Prove that if any two quantities be added together and their sum divided by the sum of their reciprocals, the quotient will be equal to the product of the two quantities.

4. If $a=pq$, $b=qr$, $c=rs$ and $a+b+c=ps$, show that $(a+b)^2+(b+c)^2=(c+a)^2$.

5. If $x+y+z=0$. show that $x(x^2-yz)+y(y^2-xz)+z(z^2-xy)=0$.

6. Prove $(a-b)\ (a+b-c)+(b-c)\ (b+c-a)=(a-c)\ (a+c-b)$.

7. If $x^2=a^2+b^2$, $y^2=c^2+d^2$, which is greater, xy or $ac+bd$, ad being not equal to bc?

8. Which is greater, n^3+1 or n^2+n, n being not $=1$?

9. Show that $\frac{1}{a}+\frac{a}{1+a}>1$ and $\frac{a}{b}+\frac{b}{a}>2$, a and b being both positive.

10. If $x=\frac{b-c}{a}$, $y=\frac{c-a}{b}$, $z=\frac{a-b}{c}$ then $xyz+x+y+z=0$·

EXERCISE XL.

1. If a, b, k be positive integers, ascertain whether $(a+k)^3-(b+k)^3$ is divisible by $a-b$.

2. Factor $4(a+b)^4+9(a-b)^4-12(a^2-b^2)^2$.

3. Factor $12x^2-31xy+20y^2+29xz-38yz+14z^2$.

4. A person bought 80 lbs. tea, some at 50c. and some at 75c. He finds by selling all at 75c. his gain would be $2.50 more than by adding 12½c. per lb. to the price of each. How much of each did he buy?

5. Prove that any trinomial is a complete square if the square of the middle term is equal to 4 times the product of the first and last terms.

6. If $16a^4+48x^3y+qx^2y^2+24xy^3+4y^4$ be a perfect square, find q.

7. Prove that $(1-2x+3x^2-4x^3, \text{etc.})\,(1+2x+3x^2+4x^3+)=(1+x^2+x^4+, \text{etc.})^2$.

8. If $s=a+\frac{1}{a}$, show that $a^4+\frac{1}{a^4}-2=s^2(s^2-4)$.

9. Find the equal factors of $x^2+4xy+4y^2-4x-8y+4$.

10. Divide $a^2(c-b)+b^2(a-c)+c^2(b-a)$ by $a-b$.

MISCELLANEOUS EXERCISES.

A.

1. Divide the product of $12a^2-11a-36$ and $28a^2-86a+66$ by $21a^2-5a-44$.

2. Divide $(2x^2+3x-1)^2-(x^2+4x+5)^2$ by the product of $(3x+4)(x+2)$.

3. If $x+y=2a$, $x-y=2b$, prove that $x^4-23x^2y^2+y^4=(7a^2-3b^2)(7b^2-3a^2)$.

4. Find the value of $x^4-47x^2y^2+y^4$ in terms of p and q when $x+y=p$ and $x-y=q$.

5. Show that the square of $x+1$ exactly divides $(x^3+x^2+4)^3-(x^3-2x+3)^3$.

6. Prove that $(x+z)^3+3(x+z)^2y+3(x+z)y^2+y^3=(x+y)^3+3(x+y)^2z+3(x+y)z^2+z^3$.

7. Find the quotient by factoring of $9a^2+6ab+b^2-4c^2-4cd-d^2$ divided by $3a+b-2c-d$.

8. Factor $x^2-2mx+m^2-n^2$.

9. Factor $(a-b)(a^2-c^2)-(a-c)(a^2-b^2)$.

10. Show that

$$\frac{(a+b)^3-c^3}{a+b-c}+\frac{(b+c)^3-a^3}{b+c-a}+\frac{(c+a)^3-b^3}{c+a-b}=2(a+b+c)^2+a^2+b^2+c^2.$$

11. Divide $\left(x^2+\frac{1}{x^2}\right)^3-8$ by $\left(x-\frac{1}{x}\right)^2$.

12. Resolve into 4 factors $(x^2-3x)^2-2(x^2-3x)-8$.

13. If $x=b+c-a$, $y=c+a-b$, $z=a+b-c$, find the value of $x^2+y^2+z^2+2xy+2xz+2yz$ in terms of a, b, c.

14. Divide $(4ac+8bd)^2-(4ad+8bc)^2$ by $(a+2b)(c-d)$.

15. Divide $(x^3-3x^2y)^2-(3xy^2-y^3)^2$ by $(x-y)^3$.

16. Find the difference between the squares of 3503 and 3497.

17. Find the algebraical expression which, divided by x^2+x-1, gives $2x^3-6x^2+8x-14$ as quotient and $22x-14$ as remainder.

18. Prove that $\frac{(m-a)(m-b)}{(m-c)(m-d)}=1$ when $m=\frac{ab-cd}{a+b-c-d}$.

19. Simplify $\dfrac{(x-y)(y-z)+(y-z)(z-x)+(z-x)(x-y)}{x(z-x)+y(x-y)+z(y-z)}$.

20. Find L.C.M. of x^3-y^3, x^3+y^3, $x^4+x^2y^2+y^4$.

21. Find the fraction in its lowest terms which is the sq. root of
$$\frac{1+4x-2x^2-12x^3+9x^4}{1-4x+6x^2-4x^3+x^4}.$$

22. Simplify $\dfrac{x^8+x^6y^2+x^2y+y^3}{x^4-y^4}$.

23. Find by factoring the sq. root of $(x^2-3x+2)(x^2-4x+3)(x^2-5x+6)$.

24. Find the co-efficient of x^4 in $(x+a)^3\times(x-a)^5$.

25. Show that $ac^3-(a^2+b)c^2+b^2$ is divisible by $ac-b$.

26. Factor $x^3-9x^2+11x+21$.

27. If $x-\frac{1}{x}=1$, show that $x^3-\frac{1}{x^3}=4$.

28. One lb. tea and three lbs. sugar cost 75c., but if sugar were to rise 50% and tea 10%, the cost would be 87½c. Find price of tea and sugar per lb.

29. Find co-efficient of x in $(x+2)(x-6)(x-10)(x+14)$.

30. Find the first four terms of $(y-y^3+y^5-y^7+\dots)^2$.

31. Simplify $\dfrac{1}{6a-18}-\dfrac{1}{6a+18}-\dfrac{1}{a^2+9}+\dfrac{18}{a^4+81}$.

32. Solve $\dfrac{5x-64}{x-13}-\dfrac{2x-11}{x-6}=\dfrac{4x-55}{x-14}-\dfrac{x-6}{x-7}$.

33. Resolve into 5 factors $x^7+x^4-16x^3-16$.

34. Multiply $(3+x-2x^2)^2-(3-x+2x^2)^2$ by $(3+x+2x^2)^2-(3-x-2x^2)^2$.

35. Divide the product of $2x^2+x-6$ and $6x^2-5x+1$ by $3x^2+5x-2$.

36. Show that $(2x-3)(x+4)$ exactly divides the difference of the squares of $3x^2+8x-25$ and $x^2+3x-13$.

37. If $x+y=m$ and $x-y=n$, then $16(x^4-7x^2y^2+y^4)=(5m^2-n^2)(5n^2-m^2)$.

38. Find the value of $x^4-2x^3y+2xy^3-y^4$ when $x=a+b$, $y=a-b$.

39. If $a+b+c=0$, show that $(2a-b)^3+(2b-c)^3+(2c-a)^3=3(2a-b)(2b-c)(2c-a)$.

40. Factor $x^3+3x^2-13x-15$.

41. What number must be added to $x^2(x+2)+7$ in order that it may be divisible by $x+4$?

42. Divide $x^2-xy+\frac{3}{16}y^2$ by $x-\frac{1}{4}y$.

43. Find the remainder when $5x^4-7x^3+3x^2-x+8$ is divided by $x-4$.

44. Factor $x^2-2xy-323y^2$.

45. A boy spent $\frac{1}{2}$ of his money for marbles, $\frac{1}{3}$ of the remainder for oranges, and $\frac{1}{5}$ of what then remained for a book, and had 12c. left; what had he at first?

46. Divide the square root of $4a^2-12ab-6bc+4ac+9b^2+c^2$ by $2a-3c$.

47. Factor $x^2+5xy-36y^2+x-4y$.

48. If $x-\frac{1}{x}=1$, prove that $x^2+\frac{1}{x^2}=3$.

49. By what quantity must $3x^2-4xy+5y^2$ be multiplied to give $18x^4+19x^2y^2+12xy^3+35y^4$?

50. Multiply $\left(x-\frac{1}{3y}\right)$ by $\left(x-\frac{1}{5y}\right)$.

51. Show that $(a+b+c)^2+a^2+b^2+c^2=(a+b)^2+(b+c)^2+(c+a)^2$.

52. Factor $9x^2-24xy-9z^2+16y^2$.

53. Factor $99x^2-4xy-143y^2$.

54. Solve $19x-21y=100$, $21x-19y=140$.

55. Find the sq. root of $(x-y)^4-2(x^2+y^2)(x-y)^2+2(x^4+y^4)$.

56. Without actual division prove that $x^5-7x^4+17x^3-22x^2+25x-18$ is divisible by $x-2$ without a remainder.

57. Find the remainder when $x^5+11x^3-5x^2+6x+13$ is divided by $x-2$.

58. What must be added to $5a^4-7a^3+6$ to make it exactly divisible by $a+3$?

59. If $a=5x-3y-2z$, $b=5y-3z-2x$, $c=5z-3x-2y$, then $a+b+c=0$.

60. Solve $\frac{x-2}{3}-\frac{y+2}{4}=0$; $\frac{2x-5}{5}-\frac{11-2y}{7}=0$.

61. Write down the quotient without actual division of $8x^3+8y^3+z^3-12xyz$ by $2x+2y+z$.

62. Factor $a^2-2b^2-6c^2+ab-ac+7bc$.

63. Find the value of $3(x+y+z)^3-(x^3+y^3+z^3)$ when $x=3$, $y=-5$, $z=7$.

64. Factor $4x^2y^2+4(a+b)xy+(a+b)^2$.

65. Factor $(x-3y)^3-(y-3x)^3$.

66. Factor $(x^2+7x+6)(x^2+7x+12)-280$.

67. Solve $\frac{12x}{x^2-9}+\frac{2}{x+3}=\frac{2}{x-3}$.

68. Simplify $\left(y-\frac{a^2-xy}{y-x}\right)\left(x+\frac{a^2-xy}{y-x}\right)+\left(\frac{a^2-xy}{y-x}\right)^2$.

69. Solve $ax+by=c$, $a^2x+b^2y=c^2$.

70. Divide a^4-b^4 by $a-b$, and from the result write down the quotient of $(a+b)^4-16c^4$ by $a+b-2c$.

71. Multiply $a^2+25b^2+4c^2+5ab-2ac+10bc$ by $a-5b+2c$.

72. Factor $x^2y^2z^2-x^2z-y^2z+1$.

73. Divide $(a+2b-3c+d)^2-(2a+b+3c-d)^2$ by $a+b$.

74. Solve $15x+17y=79$, $17x+15y=81$.

75. Solve $\frac{4x+18}{2x+4}+\frac{3x-2}{x+3}=\frac{10x+28}{2x+8}$.

76. Factor $x^2-y^2-3x-y+2$.

77. Show without actual division that $(6x^2-4x+2)^3-(4x^2+6x-10)^3$ is exactly divisible by $x-2$ or $2x-6$.

78. Show that $(1-x)^2$ is a factor of $1-x-x^5+x^6$.

79. If $2(a^2+b^2)=(a+b)^2$, show that $a=b$.

80. Show that $m(m+n)(m+2n)(m+3n)+n^4$ is a perfect square.

81. Find a number such that if $\frac{3}{8}$ of it be subtracted from 20, and $\frac{5}{11}$ of the remainder from $\frac{1}{4}$ of the original number, 12 times the second remainder will be $\frac{1}{2}$ of the original number.

82. Factor $6x^2-6y^2-20z^2+22yz+7xz-5xy$.

83. Which factor $x-\frac{y}{3}$ or $x+\frac{y}{2}$ is likely to belong to $x^3-\frac{x^2y}{2}-\frac{xy^2}{4}+\frac{y^3}{8}$, and why?

84. Find by factoring the H.C.F. of $x^3-8x^2+19x-14$ and $x^4-7x^3+8x^2+28x-48$.

85. Write down the co-efficient of x in $(x^2-21x-13)(x^2-2x-1)$.

86. Write down the co-efficient of x^4 in $1-2x+4x^2-8x^3+16x^4$ multiplied by $1+2x+4x^2+8x^3+16x^4$.

87. Write down without dividing the quotient of $x^4-5x^2y^2+4y^4$ by $x^2-3xy+2y^2$.

88. Factor $ab(a+b)+bc(b+c)+ca(a+c)+2abc$.

89. A number consists of two digits and another is formed by reversing the digits. If the sum of the two numbers is 99 and the difference 45, find the digits.

90. Factor $x^4-x^3-8x^2+7x+7$.

91. Solve $x+2y+3z=4$, $x+3y+2=4z$, $x+2z+3=4y$.

92. Find value of $\frac{a-2x}{b-2x}$ when $x=\frac{ab}{a+b}$.

93. Solve $49x+37y=1230$, $37x+49y=1350$.

94. Simplify $x^{a+b+c}\times x^{a+b-c}\times x^{a-b+c}\times x^{b+c-a}$.

95. If $a=y+z-2x$, $b=z+x-2y$, $c=x+y-2z$, find the value of $b^2+c^2+2bc-a^2$.

96. Find the remainder when $a^7-9a^4+11a^2-7$ is divided by $a-4$.

97. Solve $\frac{x}{a}+\frac{y}{b}=3$; $\frac{y}{b}+\frac{z}{c}=5$; $\frac{x}{a}+\frac{z}{c}=4$.

98. A starts from London and travels $1\frac{2}{5}$ miles per hour, B starts 8 hours after in the same direction at $1\frac{2}{3}$ miles per hour; how far will he travel before he overtakes A?

99. Factor $(x^2+y^2)^3-8x^3y^3$.

100. Factor $6x^2-13xy+6y^2+5x-5y+1$.

MISCELLANEOUS EXERCISES.

B.

1. Factor $(bc+ca+ab)^2-(b^2c^2+c^2a^2+a^2b^2)$.

2. What is the least multiplier that will make x^3-5x^2+5x-1 a multiple of x^2-4x+3?

3. Solve $\frac{2x+3}{x+1}=\frac{4x+5}{4x+4}+\frac{3x+3}{3x+1}$.

4. $44a^4-83a^3-74a^2+89a+56$ is the product of two factors, and one of them is $11a^2-7a-8$, what is the other?

5. A man has two farms rented at $\$7\frac{1}{2}$ per acre and his total rent is \$3,375. If the rent of the first was reduced to $\$6\frac{1}{4}$, and of the 2nd to \$5.00 per acre, his rent would be \$2,500. How many acres in each farm?

6. Factor $x^2+3x+1+2xy+1+3y+y^2$.

7. Factor $x^4+2a^2x^2-x^2+a^4+6x-9$.

8. Find co-efficient of x^3 in $(x-1)(x-2)(x-3)(x-4)(x-5)$.

9. Resolve $(x-1)(x-3)-(x-1)^3$ into factors.

10. Factor $(a^2+b^2+1+ab+a+b)^2-(ab+a+b)^2$.

11. Extract the sq. root of $x^{12}-6x^{10}+13x^8-14x^6+10x^4-4x^2+1$.

12. Find H.C.F. of $3x^3-13x^2+23x-21$ and $6x^3+x^2-44x+21$ and what value of x will make both vanish?

13. Divide $1+\frac{1}{a}$ by $1-\frac{1}{a^2}$.

14. Divide $\frac{x^3+y^3}{x^5-xy^4}$ by $\frac{x^2+2xy+y^2}{x^3+xy^2}$ giving the quotient in its simplest form.

15. Find the remainder when the divisor is $x+1$ and the dividend is the product of $(x+3)(x+4)(x-7)$.

16. Solve $\frac{1}{x+2}+\frac{1}{x+10}=\frac{1}{x+4}+\frac{1}{x+8}$.

17. Extract the sq. root of $(6a^2+a-2)(3a^2-7a-6)(2a^2-7a+3)$.

18. Divide 20 into two parts so that the square of the greater shall exceed the square of the less by 80.

19. Find H.C.F. of $6x^4+26x^3+15x^2-16x-10$ and $30x^4+136x^3+95x^2-79x-65$.

20. Resolve $x^5 - 4x^3 + 5x^2 - 20$ into 3 factors.

21. A dealer adds 20% to the cost of an article to make the selling price, but he gives a customer 10% discount from the selling price, and then has a profit of 75c. Find the cost price.

22. Find the value of $\frac{2a}{a-b} - \frac{2b}{a+b} + \frac{a^2-2b^2}{a^2-b^2} - \frac{3a^2}{a^2+b^2}$.

23. Solve $\frac{6x-7}{9x+6} - \frac{5x-5}{12x+8} = \frac{1}{12}$.

24. Solve $3x - 2y = 13$, $3y - 2z = 16$, $3z - 2x = 9$.

25. Extract the square root of $9x^6 - 12x^5 + 22x^4 + x^2 + 12x + 4$.

26. Divide $\frac{1}{1-a} + \frac{1}{1+a}$ by $\frac{1}{1-a} - \frac{1}{1+a}$.

27. Express $(x^3 - 3x^2)^2 - (3x - 1)^2$ as the product of 5 integral factors.

28. Simplify $\frac{m^3 - 6m^2 + 12m - 8}{m^3 - 2m^2 - 4m + 8}$.

29. Find L.C.M. of $a^2 + 6a + 5$ and $a^3 - a$.

30. Factor $2abc + a^2(b + c) + b^2(c + a) + c^2(a + b)$.

31. A boy plucks from a tree a certain number of plums, another $\frac{4}{3}$ as many. They both have 5 times as many as a third takes. All have 84. How many has each?

32. Write down the quotient without actual division of $x^4 - 4x^2y^2 + 4y^4 - x^2 - 6xy - 9y^2$ by $x^2 - x - 2y^2 - 3y$.

33. Solve $x - y + z = 5$, $3x + 4y - 5z = 13$, $x + \frac{y}{2} + \frac{z}{3} = 14$.

34. Solve $\frac{x^2 - 3x - 9}{x - 5} + \frac{x^2 - 7x - 17}{x - 9} = \frac{x^2 - 6x - 15}{x - 8} + \frac{x^2 - 4x - 11}{x - 6}$.

35. Factor $20x^2 - 11x - 42$.

36. Find the remainder after dividing $x^4 - 3x + 7$ by $x - 2$.

37. Divide $(4x - 4y + 7z)^2 - (3x - 10y - 7z)^2$ by $7(x - 2y)$.

38. Write down the square root of $(x^2 - 7)^2 + 24x(x^2 - 7) + 144x^2$.

39. Simplify $\left(\frac{a^2 + b^2}{2ab} + 1\right)\left(\frac{ab^2}{a^3 + b^3}\right) \div \frac{4a(a + b)}{a^2 - ab + b^2}$.

40. Multiply $x^2 + 2ax + 2bx + a^2 + 2ab + b^2$ by $x + a + b$.

41. A roll of cloth was bought at 66c. per yd., and another roll 25 yds. longer at 60c. per yd., the two together cost $241.80. How many yds. in each roll?

42. Find the value of $\frac{x^5-x^4y+x^3y^3}{y^5-y^4x+y^3x^3}$ when $x=2y$.

43. Solve $3x-y+2z=11$, $3y-z+2x=9$, $3z-x+2y=16$.

44. Express in words the following algebraical expression : $y(y-1)(y-2)(y-3)+1=(y^2-3y+1)^2$.

45. Simplify $\frac{x-1}{(x+2)(x+5)}-\frac{2(x+2)}{(x+5)(x-1)}+\frac{x+5}{(x-1)(x+2)}$.

46. Solve $\frac{1}{3}(2x+5)+\frac{1}{5}(2x-5)=\frac{1}{4}(3x+1)+\frac{1}{7}(3x-1)$.

47. Solve $y=3(x+1)$, $4x=y+1$.

48. The product of two numbers is 75, and the quotient of the sum by the difference is 4 times the quotient of the difference by the sum ; find the numbers.

49. Simplify $\frac{a^2-2a-3}{a^2-4a+3}+\frac{5a^2+5a-30}{a^2+2a-3}+\frac{a^2+a-2}{a^2-2a+1}$.

50. Show that $(x+y+z+a)^2-(x-y-z+a)^2=4(x+a)(y+z)$.

51. Show that $\{ab(b-a)+ac(a-c)+bc(c-b)\} \div (b-c)=(a-b)(c-a)$.

52. Show by factoring that $\frac{ab+2a^2-3b^2-4bc-ac-c^2}{2a+3b+c}=a-b-c$.

53. Show that $\frac{12a^2+7ab-12b^2+ac-7bc-c^2}{3a+4b+c}=4a-3b-c$.

54. Factor $x^2-2xy-3y^2+4yz-z^2$.

55. Simplify $(4x+5y+z)^3-3(4x+5y+z)^2z+3(4x+5y+z)z^2-z^3$.

56. Find L.C.M. of $a(x+1)$, $b(x+1)(x-1)$, $c(x^2+2x-3)$, $d(x^2+4x+3)$.

57. Solve $\frac{x-a}{bc}+\frac{x-b}{ac}+\frac{x-c}{ab}=2\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$.

58. A man bought sheep for \$528, and having lost 10, and sold 20 that were diseased at \$1.20 per head less than cost, he disposed of the rest for \$464, thereby realizing his outlay. How many did he buy ?

59. Factor $42bc+4a^2-49b^2-9c^2$.

60. Simplify $\frac{3}{1-2x}-\frac{7}{1+2x}-\frac{20x-4}{1-4x^2}$.

61. Find the value of $\dfrac{a-\dfrac{a-b}{1+ba}}{1+\dfrac{a(a-b)}{1+ab}}$.

62. Solve $a(x+y)+b(x-y)=2a$, $y(a+b)-x(a-b)=2b$.

63. Show that the sum of the cubes of $2x-3y$ and $2x+7y$ is divisible by $4(x+y)$.

64. Factor $(a^2+a-2)x^2+(2a^2+a+3)x+a^2-1$.

65. Solve $\frac{12x+10a}{3x+a}+\frac{28x+117a}{2x+9a}=18$.

66. At an examination A got 3 marks above 50% and B got 6 less than $33\frac{1}{3}$%, and A's marks were twice B's, how many had each?

67. If the dividend be $3x^4-4x^3+8x^2-7x+4$ and remainder $34x-30$ and quotient $3x^2+5x+17$, find the divisor.

68. Divide $(pq+rs)^2-(ps+qr)^2$ by $(p-r)(q-s)$.

69. Find H.C.F. of $3x^4+5x^3y+9x^2y^2+xy^3+6y^4$ and $2x^4+5x^3y+5x^2y^2-3xy^3-9y^4$.

70. Show that $\left(\frac{y^2-y+1}{12}\right)^3-27\left\{\frac{(y+1)(2y-1)(y-2)}{432}\right\}^2=\frac{y^2(y-1)^2}{256}$.

71. Simplify $(x-a)^2-(x-b)^2-(a-b)(a+b-3x)$.

72. Prove that $(a+5)^3-(a+2)^3=9(a+5)(a+2)+27$.

73. Solve $x-11y=1$, $111y-9x=99$.

74. Simplify $\frac{(m-a)(n-a)}{(a-b)(a-c)}+\frac{(m-b)(n-b)}{(b-c)(b-a)}+\frac{(m-c)(n-c)}{(c-a)(c-b)}$.

75. Express in factors L.C.M. of $1-8x+17x^2+2x^3-24x^4$ and $1-2x-13x^2+38x^3-24x^4$.

76. Find H.C.F. of $6x^3-11x^2-37x-20$ and $2x^3-4x^2-13x-7$.

77. Simplify $\frac{36x^4-13x^2+1}{30x^3-19x^2+1}$.

78. The sum of two numbers is 5760 and the difference is $\frac{1}{3}$ of the greater. Find the numbers.

79. Show that $\frac{12x+10a}{3x+a}+\frac{117a+28x}{9a+2x}=18$ when $x=3a$.

80. Factor $40x^2+61xy-84y^2$.

81. Find by factoring the H.C.F. of $2a^2-21b^2-45c^2+ab+62bc+ac$ and $3a^2-21b^2-45c^2-2ab+62bc+6ac$.

82. Divide $(22x+3y-z)^2-(17x-2y+3z)^2$ by $5x+5y-4z$.

83. Find the remainder after dividing x^3-5x^2+7x-9 by $x+3$.

84. Factor $45x^2 - 77x - 3408$.

85. Divide the square root of $x^8 + 18x^6 + 117x^4 + 324x^2 + 324$ by $x^2 + 3$.

86. Find L.C.M. of $8x^3 + 27$, $16x^4 + 36x^2 + 81$ and $6x^2 - 5x - 6$.

87. If $4x - 2y = x + 2y - 1$, show that $x^2 + y^2 = 2xy + 4$.

88. Write down the product of $(x - 3y - 5z)(2x - 3y + z)$.

89. Factor $8x^2 + 18xy - 5y^2 - 2x - 38y - 21$.

90. Divide $a^2 + (2ac - b^2)x^2 + c^2x^4$ by $a - bx + cx^2$.

91. How much greater is the co-efficient of x in the product of $(x+1)(x+2)(x+3)(x+4)$ than in that of $(x+2)(x+3)(x+4)$?

92. Find the value of $25a^2 + (a + 4b)^2$ when $3a + 2b = 7$, $a + b = 2$.

93. Find the value of $x^4 - 11x^3 - 11x^2 - 13x + 11$ when $x = 12$.

94. Factor $56x^2 + 36xy - 20y^2 + 28x - 10$.

95. By what must $a^4 + a^3b + a^2b^2 + ab^3 + b^4$ be multiplied that the product may be $a^5 - b^5$?

96. Show that $(x^2 + 6xy + 4y^2)^5 + (x^2 + 2xy + 4y^2)^5$ is exactly divisible by $x^2 + 4xy + 4y^2$.

97. Solve $5x + 2y - 1 = 3x - y + 14 = x + 19y + 6$.

98. Write down the quotient of the sum of the cubes of $a + b$ and $c + d$, by $a + b + c + d$.

99. Solve $4x - 6y - 3 = 7x + 2y - 4 = 3y - 2x + 24$.

100. A farm was rented, part at \$5.00 and part at \$8.00 per acre for \$680, but if the rates had been interchanged the amount would have been \$620. How many acres in the farm?

MISCELLANEOUS EXERCISES.

C.

1. Write down the quotient without actual division of $(x+y)^3+3(x+y)^2z+3(x+y)z^2+z^3$ by $(x+y)^2+2(x+y)z+z^2$.

2. Find the factors of the quotient of $84x^4-55x^3y-320x^2y^2-66xy^3+105y^4$ divided by $7x^2+3xy-3y^2$.

3. Solve $\frac{x+1}{3}-\frac{x-1}{4}+4x=12+\frac{2x-1}{6}$.

4. Simplify $a^2+b^2+c^2-3ab-bc-2ac+a(a+b+c)-(b-c)^2$ and divide result by $a-b$.

5. Factor $16(x^2+y^2)^2+40(x^2+y^2)z^2+25z^4$.

6. Without actual division find the remainder when x^5-5x^2+5 is divided by $x-5$.

7. Write down quotient and remainder of $\frac{x^{11}+y^{11}}{x-y}$.

8. Simplify $\frac{(x^a)^3\,(x^b)^3\,(x^c)^3}{(x^{b+c})\,(x^{c+a})\,(x^{a+b})}$.

9. Divide $(x^5-1)-5(x-1)$ by $(x-1)^2$.

10. Divide $(x+1)\,(x+2)\,(x+3)+6$ by $x+4$.

11. Solve $\frac{x}{a^2+c^2}+\frac{x}{b^2+d^2}=\frac{a^2+b^2+c^2+d^2}{(ab-cd)^2+(ad+bc)^2}$.

12. Find the remainder when $2x^5-2x^4+3x^3-7x^2+5x-8$ is divided by $x+2$.

13. Find the square root of $67x^2+9x^4-70x-30x^3+49$.

14. Expand in consecutive powers of x the expression $(1-x+x^2)(1-x^3+x^6)$.

15. What expression must be added to $x^3+11x^2+21\,(x+1)$ that it may be exactly divisible by $x+6$?

16. Find all the factors of $(a^2-3a)^2-2a^2+6a-8$.

17. Multiply $x(4x+3y)-(x+2y)^2$ by $4x(x+y)-y(11x-3y)$ and divide the product by $(3x-4y)\,(4x-3y)$.

18. Find co-efficient of x^3 in $(x+2)\,(x-3)\,(x-7)\,(x+8)\,(x-9)$.

19. Factor $(1+x)^3(1+y^3)-(1+y)^3(1+x^3)$.

20. Simplify $\frac{9x^4+18x^3-x^2+4}{3x^4+x^3+2x^2-x+1}$.

21. Simplify $\frac{1}{(x-1)^2}-\frac{1}{(x+1)^2}-\frac{4x(x^2+2)}{x^6-1}$.

22. Solve $7x+9y=71$, $9x-7y=17$.

23. Multiply $x^4+4x^3+6x^2+4x+1$ by x^3-3x^2+3x-1.

24. If $3p=x+y+z$, show that $(p-x)^3+(p-y)^3+(p-z)^3=3(p-x)(p-y)(p-z)$.

25. Factor $6x^2+15x+9$.

26. Express as a single fraction $\frac{x^2-4x+3}{x^2+2x-3}+\frac{x^2-6x+5}{x^2+4x-5}+\frac{x^2-8x+7}{x^2+6x-7}$.

27. Factor $(x-3)(x+1)^3+(x-3)(x+2)^3$.

28. Divide $x^7+x^5+x^4+x^3+x^2+1$ by $x^5+x^4+x^3+x^2+x+1$.

29. The banker's discount on a sum of money at 5% per annum is equal to the true discount on a sum $50 larger. Find the sum.

30. Factor $2xy+7x+6y+21$.

31. Factor $x^4-1-4(x-1)$.

32. Factor $(x+y)^3+2xy(1-x-y)-1$.

33. Divide $x^{10}+x^5+1$ by x^2+x+1.

34. Find the co-efficient of x^4 in $(1-x+x^2-x^3)^2$.

35. Write down the remainder of $2a^4+3a^3+4a^2+5a+6$ divided by $a-3$.

36. Show that $\frac{a+b-c}{a-b+c}-\frac{a-b+c}{a+b-c}-\frac{4(b-c)^2}{a^2-(b-c)^2}=\frac{4(b-c)}{a+b-c}$.

37. Expand in powers of x, $(1-x)^3(1+x^4)$.

38. Simplify $\frac{65x^2y^2-(x^4+64y^4)}{x^2-7xy-8y^2}$.

39. Find co-efficient of x^4 in $(1+x)^7$.

40. Find the remainder when $x^4+px^3+qx^2+rx+s$ is divided by $x-a$.

41. Factor $(x^2+x)^2+4(x^2+x)-12$.

42. Reduce to its lowest terms $\frac{x^4+ax^3-9a^2x^2+11a^3x-4a^4}{x^4-ax^3-3a^2x^2+5a^3x-2a^4}$.

43. Factor $a^4-14a^2b^2+b^4$.

44. Subtract $(x^2-5)^2$ from $(x-3)(x-1)(x+3)(x+1)$.

45. Show by factoring that $3\ x+y)^2+2(x+y)(z+u)-5(z+u)^2$ is divisible by $x+y-z-u$ and write down quotient.

46. Show without dividing that $(1+x+3x^2+3x^3)^3+(1-x+3x^2-3x^3)^3$ is divisible by $1+3x^2$.

47. Employ detached co-efficients to divide $x^5-3x^4+x^3+x^2-3x+1$ by $x+1$.

48. Solve $(a+x)^3+(b+x)^3+(c+x)^3=3(a+x)(b+x)(c+x)$.

49. Solve $\frac{4x^3+4x^2+8x+1}{2x^2+2x+3}=\frac{2x^2+2x+1}{x+1}$.

50. Find the remainder without division when $x^3-7x^2a+8xa^2+15a^3$ is divided by $x+2a$.

51. Find the continued product of a^2+a+1, a^2+a-1 and $a^4-2a^3+a^2+1$.

52. Simplify $\frac{x^4+a^2x^2-b^2x^2-a^2b^2}{x^4+a^2x^2+a^4}\div\frac{x^4-a^4}{x^3+a^3}$.

53. Simplify $\frac{(2x^2+5x+2)(x^3-3x^2-x+3)}{(x^3+6x^2+11x+6)(2x^2-x-1)}$.

54. If $(a^2+b^2)(x^2+y^2)=(ax+by)^2$ prove that $\frac{x}{a}=\frac{y}{b}$.

55. Show that $(4x+7y)^4-(3x+8y)^4$ is divisible by $7x+15y$ or $x-y$.

56. Resolve $16x^7-81x^3-16x^4+81$ into five factors.

57. If $x=b+c-2a$, $y=c+a-2b$, $z=a+b-2c$, find the value of $x^3+y^3+z^3-3xyz$.

58. Show that $(a+b)^2+(a+c)^2+(a+d)^2+(b+c)^2+(b+d)^2+(c+d)^2=(a+b+c+d)^2+2(a^2+b^2+c^2+d^2)$.

59. Prove that $(a+3b)^3+(b-4c)^3+3(a+3b)(b-4c)(a+4b-4c)=(a+4b-4c)^3$.

60. Solve $5x+2y+3z=13$, $3x+7y-z=2$, $x-2y+z=5$.

61. A man is able to pay his creditors 25c. in the \$; but if his assets were 5 times as much and his debts $\frac{2}{3}$ of what they are, he would have a balance of \$1,400. How much does he owe?

62. Write the co-efficient of x^3 and x^5 in the product of $3x^4-8x^3+5x^2-11x+13$ and $x^5+9x^4+7x^3-11x^2-8x+2$.

63. If $x+\frac{1}{x}=y$, find the value of $x^3+\frac{1}{x^3}$.

64. Find the difference of the squares of the highest and lowest of any three consecutive numbers in terms of the middle number.

65. Simplify $\dfrac{x^3-\frac{3x}{4}+\frac{1}{4}}{(2x-1)^2}$.

66. Factor $16x^2-16y^2-2x^5+2x^3y^2-3xy(8-x^3)$.

67. Solve $\dfrac{4x-34}{17}-\dfrac{258-5x}{3}=\dfrac{69-x}{2}$.

68. Solve $\dfrac{x+1}{x-1}-\dfrac{x-9}{x-7}=\dfrac{x}{x-2}-\dfrac{8-x}{6-x}$.

69. Simplify $\left\{1-\dfrac{4}{x-1}+\dfrac{12}{x-3}\right\}\left\{1+\dfrac{4}{x+1}-\dfrac{12}{x+3}\right\}$.

70. Divide $(x^2-p^2)(x^4+p^2x^2+p^4)$ by $(x-p)(x^2+px+p^2)$.

71. Resolve $64x^6y^2-y^2-256x^8+4x^2$ into 6 factors.

72. Find the product of $(x^3+3x^2+5)^2-(x^3+3x^2-5)^2$ and $(x^3+x-3)^2-(x^3-x+3)^2$.

73. Find the co-efficient of x^5 in the product of $1-2x+x^2$ and $1+2x+3x^2+4x^3+5x^4$.

74. A's money, twice B's and 3 times C's = $190
B's " " C's " 3 " A's = 175
C's " " A's " 3 " B's = 175.
How much had each?

75. Extract the square root of $x^2+(1+x^2)(1+x)^2$.

76. Show that $(x+a)(x+b)(x+c)=(x-a)(x-b)(x-c)+2\{(a+b+c)x^2+abc\}$.

77. Multiply $y-\dfrac{x^2}{y}$ by $\dfrac{y}{x}+\dfrac{x}{y}$.

78. Find the sq. root of $(2x+1)(2x+3)(2x+5)(2x+7)+16$.

79. Solve $\dfrac{6x^2-7x+2}{2x-1}+\dfrac{15x^2-17x+4}{3x-1}=\dfrac{28x^2-27x+5}{4x-1}$.

80. If $x^4+8x^3-ax^2-168x+441$ is an exact square, find a.

81. Divide $(ac+bd)^2-(ad+bc)^2$ by $(a-b)(c-d)$.

82. Show that $a^2+ab+b^2-a(a-c)+bc=(a+b)(b+c)$.

83. Solve $\dfrac{3x^2-4x+5}{3x-4}=\dfrac{4x^2-5x+7}{4x-5}$.

84. A number consists of two digits. If the left hand digit be increased by 5 and the right diminished by 5 the original number will be doubled. If these digits be transposed the result will be $\frac{1}{5}$ of the original number. Find the number.

85. Prove that half the sum of the squares of

$$\frac{x+a}{x-a} \text{ and } \frac{x-a}{x+a} = \frac{8a^2x^2}{(x^2-a^2)^2} + 1.$$

86. Simplify $\dfrac{\frac{a}{b}+2+\frac{b}{a}}{a+b} + \dfrac{\frac{a}{b}+\frac{b}{a}-2}{a-b}$.

87. If $9x^4+24x^3y+nx^2y^2-8xy^3+y^4$ be a perfect square, find n.

88. Find what value of d will make $a-b+c-d+bc-ad+bd-ac+a^2+b^2+c^2$ vanish when $4a+5b=c$, $3a+4b+1=0$, $b+c+4=0$.

89. Factor $108x^2-1383xy+4277y^2$.

90. Factor $4(x+2)^4-37x^2(x+2)^2+9x^4$.

91. Find the H.C.F. of $21x^2+38x+5$ and $129x^2+221x+10$.

92. Prove that $(x+3)^3-(x+2)^3$ is equal to $3x^2+15x+19$.

93. Divide by factoring $a^2(b+c)+b^2(c+a)+c^2(a+b)+2abc$ by $ab+ac+b^2+bc$.

94. Find value of $3x^5-160x^4+344x^3+700x^2-1910x+1200$ when $x=51$.

95. Simplify $\{(a+3b)^2+2(a+3b)(a-b)+(a-b)^2\} \{a-b\}^2$.

96. Simplify $\dfrac{a^4+x^4+ax(a^2+x^2)+a^2x^2}{a^5-x^5} \div \dfrac{a^2+ax+x^2}{a^3-x^3}$.

97. Write down the product of $6x^3+13x^2-4x-15$ by $6x^3-13x^2-4x+15$.

98. Find H.C.F. of $1-x+y+z-xy+yz-xz-xyz$ and $1-x-y-z+xy+yz+xz-xyz$.

99. Factor $12a^2-132ab-143ac-156bc-144b^2-12c^2$.

100. Solve $\dfrac{4x+81}{10y-17}=6$, $\dfrac{12x+97}{15y-17}=4$.

MISCELLANEOUS EXERCISES.

D.

1. If $ps=qr$, show that $\frac{p^3r}{qs}+\frac{2pr^2}{s}+\frac{prs}{q}=\left(\frac{p^2}{q}+r\right)^2$.

2. Find the relation that must exist among a, b, c so that $x^4+ax^3+bx^2+cx+1$ may be a complete square as regards x.

3. Find the value of x that $8x^3-36x^2+56x-39$ may be a complete cube.

4. Find the relation between b and c that x^3+3ax^2+bx+c may be a perfect cube for all values of x.

5. Show that $(a^2+b^2+c^2)(x^2+y^2+z^2)-(ax+by+cz)^2=(bz-cy)^2+(cx-az)^2+(ay-bx)^2$.

6. Under what conditions is $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{a+b+c}$, when a, b, c are not each equal to zero?

7. Solve $\frac{13x-10}{36}+\frac{4x+9}{18}-\frac{7(x-2)}{12}=\frac{13x-28}{17x-66}$.

8. Find 4 values of a for which $6x^2+ax-35$ is resolvable into factors of the first degree in x whose co-efficients are integral numbers. State how many more could be found by your method.

9. Find the values of c and d so that $x^4+12x^3+8x^2+cx+d$ may be the square of an expression in the form of x^2+px+q.

10. Find the value of p and q if $9x^6-12x^4y^2+30x^3y^3+4x^2y^4-pxy^5+qy^6$ is a perfect square.

11. If $x=a+d$, $y=b+d$, $z=c+d$, then $x^2+y^2+z^2-xy-xz-yz=a^2+b^2+c^2-ab-ac-bc$.

12. Show that if any integer be put for x in the expression $x^6-4x^5+14x^4-32x^3+49x^2-60x+36$ the result will be a square number.

13. If $(b+c)x=a$, $(c+a)y=b$, $(a+b)z=c$, prove $xz+yz+xy+2xyz-1=0$.

14. Write down the quotient of $x^6-3x^5y+3x^4y^2-x^3y^3-x^3-8-6x^2-12x$ by $x^2-x-xy-2$.

15. Extract the sq. root of $\left(y+\frac{1}{y}\right)^2-4\left(y-\frac{1}{y}\right)$.

16. Prove the following identity:

$$\frac{1}{\left(1-\frac{b}{a}\right)\left(1-\frac{c}{a}\right)}+\frac{1}{\left(1-\frac{a}{b}\right)\left(1-\frac{c}{b}\right)}+\frac{1}{\left(1-\frac{a}{c}\right)\left(1-\frac{b}{c}\right)}=1.$$

17. Extract the sq. root of $(m-1)(m-3)(m-5)(m-7)+16$.

18. Resolve $9a^2(x^3+12ab^2)-(4b^2x^3+243a^5)$ into four factors.

19. Show that $(m+n+p+q)^2=(m+n)^2+(m+p)^2+(m+q)^2+(n+p)^2+(n+q)^2+(p+q)^2-2(m^2+n^2+p^2+q^2)$.

20. Solve $\frac{1}{ax}+\frac{1}{bx}+\frac{1}{cx}=\frac{1}{2}(a+b+c)^2-\frac{1}{2}\left(\frac{a}{bcx}+\frac{b}{acx}+\frac{c}{abx}\right)$.

21. Solve $\frac{1-ax}{bc}+\frac{1-bx}{ac}+\frac{1-cx}{ab}=\left(\frac{2}{a}+\frac{2}{b}+\frac{2}{c}\right)x$.

22. Find value of $\left(\frac{1}{11}-\frac{12}{x+6}+\frac{10}{x+5}\right).\left(\frac{1}{11}+\frac{12}{x-6}-\frac{10}{x-5}\right)$.

23. Find sq. root of $\left(x^2+\frac{x^2}{y}-2x\right)\left(1+\frac{1}{y}\right)+1$.

24. Simplify $2\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)-\frac{b+c-a}{bc}-\frac{c+a-b}{ca}-\frac{a+b-c}{ab}$.

25. Show that $(x+y)^4+(x-y)^4-2(x^2-y^2)^2=16x^2y^2$.

26. Find the sq. root of $x^4-x^3+\frac{x^2}{4}+4x-2+\frac{4}{x^2}$.

27. From the sum of the extremes of $\frac{1}{x-3a}$; $\frac{1}{x-a}$; $\frac{1}{x+a}$; $\frac{1}{x+3a}$ take the sum of the means.

28. What quantity must be added to $9x^4-6x^3+43x^2-14x+25$ to make it a complete square?

29. Factor $(a^2+4a+4)x^2+(2a^2+a-6)x+a^2-3a+2$.

30. If $2x^4-10x^3y+25x^2y^2-Rxy^3+20y^4$ is divisible by $x^2-3xy+4y^2$ without a remainder, find R.

31. Factor $(3a^2-5a-2)x^2+(4a^2+5a+2)x+a^2+2a$.

32. Factor $x^6-y^6+2xy(x^4+x^2y^2+y^4)$.

33. Find the value of $4x^5+9x^3-5x^2+23x+6$ if $2x^2=3x-4$.

34. Show that $2(a^2+b^2+c^2+ab+ac+bc)-(ab+ac+bc)=(a+b+c)^2+\frac{a^3+b^3+c^3-3abc}{a+b+c}$.

35. Show that $(2x+3y)^2+(2y+3z)^2+(2z+3x)^2+2(2x+3y)(2y+3z)+2(2y+3z)(2z+3x)+2(2z+3x)(2x+3y)=25(x+y+z)^2$.

36. Simplify

$$\left(\frac{a}{b}+\frac{b}{a}\right)^2+\left(\frac{b}{c}+\frac{c}{b}\right)^2+\left(\frac{c}{a}+\frac{a}{c}\right)^2-\left(\frac{a}{b}+\frac{b}{a}\right)\left(\frac{b}{c}+\frac{c}{b}\right)\left(\frac{c}{a}+\frac{a}{c}\right).$$

37. Solve $\frac{6}{2-3x}+\frac{10}{6-5x}+\frac{4}{10+x}=0$.

38. Find the value of a in the expression $100x^2+80x+a$ so that one of the factors of the expression may be 4 times the other and the sum of the factors is $25x+10$.

39. Extract the sq. root of $16x^2-96x+216-\frac{216}{x}+\frac{81}{x^2}$.

40. Solve $\frac{x^2-2x}{x^2-3x+2}-\frac{x^2-5x+4}{x^2-6x+8}=\frac{x^2+2x-8}{x^2+x-12}-\frac{x^2+x-12}{x^2-16}$.

41. Find an expression containing no higher power of x than the first which added to $x^4+6x^3+12x^2+6x+1$ will make it a complete square.

42. Find the value of a when the fraction $\frac{x^3+(2a-1)x^2+x+1}{x^3+(2a+1)x^2+3x+1}$ admits of reduction, and reduce it.

43. Obtain an expression which will divide both $4x^2+ax-10$ and $4x^3+bx^2-3x-15$, if $b=2a+1=7$.

44. Find what values of m will make $3mx^2+(6m-12)x+8$ a perfect square.

45. Solve $\frac{x^4+2x^3+x^2-7x-3}{x^2+3x+5}=\frac{x^4+6x^3+2x^2-16x-4}{x^2+7x+10.}$

46. Simplify $\frac{(x-1)^4+x^4+1}{(x-1)^2+x^2+1}$.

47. If $(a+b)^2+(b+c)^2+(c+d)^2=4(ab+bc+cd)$ then $a=b=c=d$.

48. Multiply and arrange in descending powers of x the expression $(1+x+x^2)\,(1+x^2+x^4)\,(1-x+x^2)$.

49. Solve $\frac{1}{39}(x+4)-\frac{1}{5}(1-x)=2-\frac{3}{26}(6-5x)-\frac{1}{5}(x+4)$.

50. Factor $x^3+(a-b-1)x^2-(a-b+ab)x+ab$.

51. Find the co-efficient of x^3 in $(x+1)^7$.

52. Find the co-efficient of x and x^3 in $(1+x+x^2)^3+(1-x+x^2)^3$.

53. When $a^2+b^2=c^2$, find the value of $(a+b+c)\,(b+c-a)\,(a+c-b)\,(a+b-c)$.

54. Simplify $\frac{20x^2+13x-21}{12x^2+19x-21}\times\frac{12x^2+x-63}{44x^2-47x-117}\times\frac{22x^2+103x+91}{10x^2+49x+49}$.

55. Find the factors of $x^{3m}-y^{3n}$ when m and n are positive integers.

56. Find the conditions that x^3+5x^2-ax+b may be divisible by both $x-2$ and $x-5$.

57. Find the conditions that x^3+7x^2+ax+b may be divisible by $x+3$ and $x-2$ for all values of x.

58. If $\frac{a+x}{a-x}+\frac{a-x}{a+x}=\frac{b+x}{b-x}+\frac{b-x}{b+x}$, prove $a=b$.

59. If $\frac{1}{x-y}+\frac{1}{x-z}=\frac{2}{x}$, prove $\frac{1}{y}+\frac{1}{z}=\frac{2}{x}$.

60. Determine which is the greater fraction, $\frac{3+m}{4+m}$ or $\frac{3-n}{4-n}$ when m is any positive number, and n any positive number less than 4.

61. What must be the value of x in order that $\frac{(a+2x)^2}{a^2+70ax+3x^2} = 1\frac{1}{3}$ when $a=67$?

62. The first two terms of a certain perfect square are $64x^4-64x^3$, and the last two terms $14x+\frac{49}{16}$. Find the square root of the expression.

63. What number must be added to the product of any four consecutive odd numbers to make it a complete square?

64. If $ab+bc+ca=0$, prove that $(a+b+c)^3=a^3+b^3+c^3-3abc$.

65. Show that $x^8+y^8+(x+y)^8$ is divisible by x^2+xy+y^2 without a remainder.

66. What is the least integral multiplier that will make $17x^5-68x^4y+102x^3y^2-68x^2y^3+17xy^4$ a complete cube?

67. Show that the product of any four consecutive integers increased by unity is a perfect square.

68. If $\frac{a}{x}(b-c)+\frac{b}{y}(c-a)+\frac{c}{z}(a-b)=0$, prove $\frac{x}{a}(z-y)+\frac{y}{b}(x-z)+\frac{z}{c}(y-x)=0$.

69. If $\frac{b^2+c^2-a^2}{2bc}+\frac{c^2+a^2-b^2}{2ac}+\frac{a^2+b^2-c^2}{2ab}=1$, show that $(a+b-c)(a+c-b)(b+c-a)=0$.

70. If $(a^2-bc)(b^2-ac)(c^2-ab)=0$, show that

$$\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}=\frac{a^3+b^3+c^3}{a^2b^2c^2}.$$

71. If $\frac{x^2-y^2}{a-b}=\frac{xy}{z}$ and $\frac{y^2-z^2}{b-c}=\frac{yz}{x}$, then $\frac{z^2-x^2}{c-a}=\frac{zx}{y}$.

72. If $3(a^2+b^2+c^2)=(a+b+c)^2$, prove $a=b=c$.

73. Find the co-efficient of x^4 in multiplying $1+2x+3x^2+4x^3+5x^4$ by $1-2x+3x^2-4x^3+5x^4$.

74. What is the least factor that will make $x^3-11x^2+40x-48$ a complete square?

75. What is the least expression used as a multiplier that will make x^3-5x^2+5x-1 a multiple of x^2-8x+7?

76. If $x^2=3x-4$, prove that $x^6+45x=44$.

77. Prove that the product of the sum of the squares of any two quantities and the sum of the squares of any other two quantities is always equal to the sum of the squares of two quantities.

78. Prove that if the sum of two numbers be multiplied by the sum of their reciprocals the product is not less than 4.

79. If $x^2y=z(x+y-z)^2$, prove $(x-z)^2=yz$.

80. If $x+y+z-xyz=2$, prove $(1-x)^2=(1-xy)(1-xz)$.

81. If $b(bx^2+a^2y)=a(ay^2+b^2x)$, prove $bx+ay=ab$ and $ay=bx$.

82. If $(a+b-c-d)x=cd-ab$, prove $(a+x)(b+x)=(c+x)(d+x)$.

83. If $(a^2-bc)x+(b^2-ca)y+(c^2-ab)z=0$ and $x+y+z=0$, prove $ax+by+cz=0$.

84. If $(2a-3y)y=(z-x)^2$ and $(2a-3z)z=(x-y)^2$ then $x+y+z=a$ and $(2a-3x)x=(y-z)^2$.

85. If $a=\frac{x-y}{x+y}$, $b=\frac{y-z}{y+z}$, $c=\frac{z-x}{z+x}$, then prove $\left(\frac{1+a}{1-a}\right)\left(\frac{1+b}{1-b}\right)\left(\frac{1+c}{1-c}\right)=1$.

86. Show that $(1+x)(1+x^2)(1+x^4)\ldots\ldots$ to n factors $=\frac{1-x^{2^n}}{1-x}$.

87. Is x^2-xy+y^2 a factor of $(x-y)^5-xy(x-y)(x^2+y^2)$?

88. Prove $(x-y)^5-x^5+y^5=5xy(x^2-xy+y^2)$.

89. If $(x-y)z^2=c^3$, $(y-z)x^2=a^3$, $(x-z)y^2=b^3$ and $(x-y)(y-z)(z-x)=3abc$, prove $a^3+b^3+c^3-3abc=0$.

90. If $x(1+y)=1$ and $y(1+z)=z$, prove $-z=1+x+2x^2+4x^3+$ etc.

91. The H.C.F. of two expressions is $a-7$, and the L.C.M. $a^3-10a^2+11a+70$, and one of the quantities is $a^2-12a+35$, what is the other?

92. Simplify $\frac{x^{m-1}y^n}{x^{2m}y^{n+1}}$.

93. Prove that if the sum of three quantities is zero, then the sum of their cubes is equal to three times their product.

94. If the sum of the cubes of three quantities be equal to three times their product, then the sum of the quantities is zero.

95. Prove that $a^2-bc+b^2-ac+c^2-ab$ is not changed by subtracting the same quantity from each a, b, c.

96. Show that the value of $x^3+y^3+z^3-3xyz$ is not changed if x^2-yz, y^2-xz, z^2-xy be substituted for x, y, z respectively.

97. Find the value of x that will make both of the following equal to zero, $x^4+3x^3+12x-16$ and $x^3-13x+12$.

98. If $a+b+c=0$, show that $\frac{a(b^3-c^3)}{b-c}+\frac{b(c^3-a^3)}{c-a}+\frac{c(a^3-b^3)}{a-b}=0$.

99. Determine the values of p and q that will make $4y^4-12y^3+py^2+qy+16$ a perfect square for all values of x.

100. If ax^2+bx+c becomes 8, 22, 42 respectively when $x=2, 3, 4$, find its value when $x=-\frac{1}{3}$.

101. Find the value of x that will make $x^4+6x^3+13x^2+13x-1$ a perfect square.

102. Determine numerical values for A, B, C, D, so that $2x^3-13x^2+26x-14=A(x-1)(x-2)(x-5)+B(x-1)(x-2)+C(x-1)+D$, may be an identity.

103. If $x+a$ is a common factor of x^2+px+q and x^2+lx+m, show that $\frac{m-q}{l-p}=a$.

104. Find H.C.F. of $1-m-m^3+m^5$ and $1-m^4-m^6-m^7$.

105. For what numerical values of p can the fraction $\frac{2px^2+(3p+4)x+7}{(x+1)(x+2)}$ be reduced to lower terms?

106. Show that $xy+xz+yz$ is a factor of $(x^2-yz)(y^2-xz)+(z^2-xy)(y^2-xz)+(z^2-xy)(x^2-yz)$ and find the other quadratic factor.

ANSWERS.

ANSWERS.

EXERCISE I.

ADDITION.

Page 1. (1.) $34a+12b-7c$. (2.) $7ax^2+21ax-2by-3y^2+5$.
(3.) $\frac{1}{12}a+\frac{43}{60}b+\frac{41}{30}c$. (4.) $2a+2b+2d$. (5.) $p+q+s$.
(6.) $3xz-5c^3$. (7.) $9a-7b+4c$. (8.) $70(abc+a^2b^2c^2)$.
(9.) $6x-3y+15z$. (10.) $2\frac{a}{b}$ ·

EXERCISE II.

SUBTRACTION.

(1.) $2a-2x+18$. (2.) $a^2-14xy+2y^3+4z^2+m$.
(3.) $16a-14c+14xy^2-13\sqrt{a-b^2}$.
(4.) $p^2-17q^2-2pq+99$.
Page 2. (5.) $a^3+a^2b+9ab^2-2b^3$. (6.) $x^2-\frac{4}{3}xy$.
(7.) $12a+10b-22c$. (8.) $x^3-6x^2y+11xy^2-6y^3$.
(9.) $a^2+{}^{a}+\frac{3}{10}$. (10.) $-\frac{a}{2}$.

EXERCISE III.

ADDITION AND SUBTRACTION.

(1.) $-4a+27c$. (2.) $2+20x+2x^2$.
(3.) $5x-3\sqrt[3]{xyz}+11y+9z$. (4.) $x^2-a^2+y^2+b^2$.
(5.) (6.) c^2. (7.) 0. (8) b^2.
(9.) $\frac{9a}{2}+2b$. (10.) $a^2+b^2+c^2$.

EXERCISE IV.

Page 3. (1.) $a-4b+3c$. (2.) $3a-2x+c+1$. (3.) $3c$. (4.) c.
(5.) $5b-5c$. (6.) $2a-b-d$. (7.) $-4b$. (8.) 0.
(9.) $6p+6q-6r$. (10.) a.

EXERCISE V.

MULTIPLICATION.

A.

(1.) $x^3-3x^2y+3xy^2-y^3+x^2-2xy+y^2$.
(2.) $x^3-y^3+z^3+3xyz$. (3.) $1-x^2-12x^4$.
(4.) $32x^5+y^5$. (5.) $a^4-2a^2b^2+b^4+4abc^2-c^4$.
(6.) $x^6-57x^4+266x^2-1$. (7.) $1-x^7$. (8.) $(x^2-a^2)^3$.
(9.) $x^{2m}+2x^my^n+y^{2n}$. (10.) a^4-b^4.

B.

(1.) $(a+b+c+d)^3$. (2.) $a^8-4a^4+16a^2-16$.
(3.) $27x^3+8y^3+z^3-18xyz$. (4.) $1+y^3+z^3-3yz$.
Page 4. (5.) $ax-a^4x^4$. (6.) $a^8-2a^4b^4+b^8$.
(7.) $x^4-y^4-z^4+4y^3z-6y^2z^2+4yz^3$. (8.) $x^{10}-1$.
(9.) $a^2-(mx-nx^2)^2$. (10.) $x^8-\frac{a^8}{256}$.

C.

(1.) $x^{12}+y^{12}$. (2.) $(b-a)$.
(3.) $(x^2+ab)^2-(ax+bx)^2$. (ii) 0.
(4.) $x^6-2x^3a^2y+a^4y^2-y^6$. (5.) $x^2-(a-b)^2$.
(6.) $x^3-(a+b+c)x^2+(ab+ac+bc)x-abc$.
(7.) Co-efficient of x above is $(ab+ac+bc) \therefore 8\times 3+8\times -2+3\times -2=2$.
(8.) $x^3-(1+2+3)x^2+(2+3+6)x-6=x^3-6x^2+11x-6$.
(9.) $x^3-15x^2+71x-105$. (10.) 1408.

EXERCISE VI.

DIVISION.

A.

(1.) $4a^2-3a+5$. (2.) $a^2-2ax+x^2$.
(3.) $x^4-x^3+x^2-x+1$. (4.) $x+b$. (5.) x^2-ax+b.
(6.) $a+b-c$.
Page 5. (7.) $8x-3y$. (8.) $x^3+6x^2+11x+6$. (9.) x^2+y^2.
(10.) $(x+a)^2-(x+a)b+b^2$.

B.

(1.) $a+b+2c$. (2.) x^3+3x^2+2.

(3.) Hint $a^3+b^3-c^3+3abc$ is divisible by $a+b-c$, $\therefore$ $(x)^3+(2y)^3-(3z)^3+18xyz$ is divisible by $x^2+2y-3z$. Result $=x^2+4y^2+9z^2-2xy+3xz+6yz$.

(4.) $(a+b)-3(c+d)$. (5.) $a+x$. (6.) $(x+b)(x+c)$.

(7.) $2a^2-3ab+4b^2$. (8.) a^4-a^2+1. (9.) $x - xy^3+y^4$.

(10.) Divide in the ordinary way, and since remainder is zero, $a=2$.

C.

(1.) $a=20$, $b=85$. (2.) $m=56$.

(3.) Apply principle $\frac{x^3+y^3}{x+y}$, etc., then $\frac{(2x+3y)^3+(2y+3z)^3}{2x+3y+2y+3z}=$ $4x^2+8xy+7y^2-6xz+3yz+9z^2$.

(4.) x^8-x^4+1.

(5.) Apply difference of squares. Result, $a-b$.

(6.) $x-b$. (7.) $b+c-a$.

Page 6. (8.) $8x^2-22ax+15a^2$. (9.) $7x+4z$. (10.) $a+b+c$.

EXERCISE VII.

MISCELLANEOUS EXERCISE.

(1.) a^2+4a+4. (2) $x^4-7x^2y^2+y^4$. (3.) $\frac{1}{2}x-y-z$.

(4.) x^4-10x^2+9. (5.) $4x^2-8x+7$.

(6.) $\frac{1}{2}-a+2a^2-4a^3$. (7.) $\frac{1}{3}+\frac{3x}{2}-\frac{x^2}{2}\cdot$ (8.) $2an$.

(9.) $a=7$. (10.) $\frac{a^2}{25}-\frac{ab}{15}+\frac{b^2}{9}\cdot$

(11.) Write 1st $(9x^2-6ax+a^2)-(x^2+4ax+4a^2)$, 2nd $(9x^2+6ax+a^2)+(x^2+4ax+4a^2)$. Sum and difference, etc.

(12.) $z^2+1+\frac{1}{z^2}\cdot$ (13.) x^2+3x+1.

(14.) Perform the division and remainder must be zero, etc., $r=aq+pb-2ab-a^2p+a^3$, $s=bq-b^2-apb+a^2b$.

(15.) $x^4+x^2+1+\frac{1}{x^2}+\frac{1}{x^4}\cdot$ (16.) $-2b$. (17.) a^6-b^6.

Page 7. (18.) $a=-1$, $b=0$. (19.) $\frac{1}{a^2}+\frac{2}{ac}+\frac{1}{c^2}-\frac{1}{b^2}\cdot$

(20.) $a=5$, $b=3$. (21). $x-\frac{1}{a}\cdot$ (22.) $m=2$.

(23.) $a = -6$, $b = 11$, $c = -6$.

(24.) Divide in ordinary way and remainder must be zero $\therefore n^2a^2 - na^2 = a^2$ or $n^2 = n + 1$.

(25.) $a^5 + 2a^4 + 4a^3 - 8a^2 - 16a - 32$.

(26.) $x^6 - 3x^5 + 9x^4 - 27x^3 + 81x^2 - 243x + 729$.

(27.) $a = -1$. (28.) 10. (29.) $x^2 + 2x + 3$.

(30.) $2x + 3$.

EXERCISE VIII.

HORNER'S METHOD OF DIVISION.

A.

(1.) $a^2 + 2a + 4$. (2.) $5x^2 + 11x + 11$.

(3.) $x^3 - 3x^2 + 3x + 1$. (4.) $3x^4 - 2x^3 - x + 5$.

(5.) $2x^2 + 3x - 1$. (6.) $x^2 - xy + y^2$. (7.) $2x^2 + x + 1$.

(8.) $x^4 - 4x^3 + 6x^2 - 4x + 1$. (9.) $x^4 - 29x^3 + 47x^2 - 25$.

(10.) $3x^3 - 2x^2 - 5x - 3$.

B.

Page 8. (1.) -7. (2.) 15. (3.) -205. (4.) -6400.
(5.) 101. (6.) 20. (7.) 1. (8.) 1. (9.) 943.
(10.) -7617. (11.) 0. (12.) 0.

EXERCISE IX.

INVOLUTION.

(1.) $a^2 + 2ab + b^2$; $4a^2 + 12ab + 9b^2$;
$\frac{x^2}{4} + \frac{xy}{3} + \frac{y^2}{9}$; $a^2 + 4ax + 4x^2$.

(2.) $a^2x^2 + 2abx + b^2$; $225x^2 + 420xy + 196y^2$;
$\frac{9a^2}{25x^2} + \frac{8}{5} + \frac{16x^2}{9a^2}$; $\frac{x^2}{y^2} + 2 + \frac{y^2}{x^2}$.

(3.) $a^2 - 2ab + b^2$; $a^2 - 4ab + 4b^2$;
$4x^2 - 24xy + 36y^2$; $\frac{4x^2}{9y^2} - 2 + \frac{9y^2}{4x^2}$.

(4.) $49x^4 - 70x^2y^2 + 25y^4$; $\frac{x^2}{y^2} - 2 + \frac{y^2}{x^2}$;
$\frac{x^2}{9} - \frac{xy}{6} + \frac{y^2}{16}$; $\frac{9x^4}{16} - \frac{6x^2y^2}{5} + \frac{16y^4}{25}$.

(5.) $a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$;
$4x^2 + 9y^2 + 16z^2 + 12xy + 16xz + 24yz$;
$1 + 2x + 3x^2 + 2x^3 + x^4$.

(6.) $16a^2+25b^2+36c^2+40ab+48ac+60bc$;
$\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{16}+\frac{xy}{3}+\frac{xz}{4}+\frac{yz}{6}$; $1+x+\frac{11x^2}{12}+\frac{x^3}{3}+\frac{x^4}{9}$.

(7.) $a^2+b^2+c^2+2ab-2ac-2bc$;
$a^2+b^2+c^2-2ab+2ac-2bc$;
$a^2+b^2+c^2-2ab-2ac+2bc$.

(8.) $x^2+\frac{1}{4}y^2+1-xy+2x-y$;
$x^4-10x^3+39x^2-70x+49$.
$x^4-2ax^3+(a^2-2b)x^2+2abx+b^2$.

(9.) $\frac{x^4}{9}+\frac{2x^3}{3}+\frac{4x^2}{3}+x+\frac{1}{4}$;
$9x^4-2x^3-17\frac{8}{9}x^2+2x+9$; $\frac{4m^2}{9n^2}+\frac{9n^2}{4m^2}-2$.

(10.) $a^2x^2+b^2y^2+c^4+2ac^2x+2abxy+2bc^2y$;
$\frac{x^2}{y^2}+\frac{y^2}{z^2}+\frac{z^2}{x^2}+\frac{2x}{z}+\frac{2z}{y}+\frac{2y}{x}$.

EXERCISE X.

Page 9. (1.) $x^3+3x^2y+3xy^2+y^3$; $x^3-3x^2y+3xy^2-y^3$; $x^3+y^3+z^3+3x^2y+3x^2z+3xy^2+3xz^2+3yz^2+3y^2z+6xyz$; $x^3+y^3-z^3+3x^2y-3x^2z+3xy^2+3xz^2+3yz^2-3y^2z-6xyz$.

(2.) $m^3+\frac{1}{m^3}+3m+\frac{3}{m}$; $m^3-\frac{1}{m^3}-3m+\frac{3}{m}$;
$\frac{m^3}{n^3}+\frac{n^3}{m^3}+\frac{3m}{n}+\frac{3n}{m}$.

(3.) $a^3-b^3+c^3-3a^2b+3ab^2+3a^2c+3ac^2+3b^2c-3bc^2-6abc$; $a^3-b^3-c^3-3a^2b+3ab^2-3a^2c+3ac^2-3b^2c-3bc^2+6abc$; $1+3x+6x^2+7x^3+6x^4+3x^5+x^6$.

(4.) $4(a+b)^2$. (5.) a^2. (6.) (7.)

(8.) $2(a-c)(b-d)$. (9.) $2(1+3a^4)$. (10.) $27x^3$.

EXERCISE XI.

MISCELLANEOUS EXERCISE.

(1.) (2.) 0. (3.)

(4.) $x^2+y^2+z^2+2xy+2xz+2yz$. (5.) $2(1+3x^4)$.

(6.) $(a+b)^3$.

(7.) Factor expression $x^3-8y^3-27y^3-18xyz$ and one factor is $x-2y-3z$, which is equal to zero, $\therefore x^3-8y^3-27z^3=18xyz$.

(8.) $=0$. (9.) $8x^3$. (10.) 0. (11.) a^3.

(12.) $8(x^2+y^2)^3$.

FACTORING.

EXERCISE XII.

Page 10. (1.) $(a+b+cd)x$; $(a+p)(x+y+z)$.
(2.) $(a-b)(x-y)(x+y)$; $(1-a)(1-b)$.
(3.) $3b^2(3a^4+11a^2b-4b^2)$; $(1-x^2)(1+x^2+p+q)$.
(4.) $3ac^2(5b^2c+4a^2b-7c^2)$; $(2a^2-1)(x^2-1)$.
(5.) $(4x+y)(a+b)$; $(a-1)(a+b)$.
(6.) $(2x+2f)(a+b)$; $(x-3)(x-y)$.
(7.) $(ax-b)(cx+d)$; $(x^2-a^2)(x^2+ax+a^2)$.
(8.) $(1-a+b)(1+p+q)$.
(9.) $(1-b)(a-b+c)$; $(a^2-1)(a+1)$.
(10.) $(a+b-c)(d-e+f)$.

EXERCISE XIII.

COMPLETE SQUARES.

A.

(1.) $(a+4b)^2$; $(a+7b)^2$. (2.) $(a+18)^2$; $(x-5a)^2$.
(3.) $(xy-8)^2$; $b(2x-5y)^2$. (4.) $(m^2n^2+1)^2$; $(4x^2+2)^2$.
(5.) $(a-9)^2$; $(1-4x)^2$. (6.) $(\frac{1}{2}x^2+2y)^2$; $(c^m-1)^2$.
(7.) $(xy^2-6)^2$; $(a+b+c+d)^2$.
(8.) $(x+y+z)^2$; $(4x^2+9y^2)^2$.
(9.) $(3a-2b+4c)^2$. (10.) $(3x+2y-z)^2$.

B.

(1.) $(\frac{1}{2}x^2-4yz)^2$; $(a-b+c)^2$. (2.) 0.
Page 11. (3.) $(2a^2-3b+4c)^2$.
(4.) $\left(\frac{2a^2}{3}+\frac{4b^2}{5}+\frac{3c^2}{4}\right)^2$.

Note.—Question should be $\frac{16a^2b^2}{15}$.

(5.) $(2b+3c-1)^2$. (6.) $\left(x-\frac{y}{2}+\frac{z}{3}\right)^2$.
(7.) $(p+q+r-s)^2$. (8.) $\left(\frac{a}{b}-\frac{b}{a}\right)^2$; $36y^2$.
(9.) $(2a^2-3a+4)^2$. (10.) $(a^2+b^2-c^2)^2$.

C.

(1.) $\{2(a+b)+3(c+d)\}^2$. (2.) $(2a-b+c)^2$.

(3.) $16x^m-14x^n$.

(4.) Multiply second expression by 2 and add to first, etc.

(5.) $a^6-a^5-\frac{47}{4}a^4+\frac{20a^3}{3}+\frac{107a^2}{3}-4a+\frac{1}{9}$.

(6.) $(3a+2)^2\,(a-3)^2\,(2a-1)^2$. (7.) $\left(\frac{a}{4}+3-\frac{2x}{3}\right)^2$.

(8.) Multiply out and re-arrange, etc.

(9.) $(x^2-2xy+y^2)^2$. (10.) $\left(\frac{a}{b}-\frac{b}{c}-\frac{c}{a}\right)^2$.

EXERCISE XIV.

DIFFERENCE OF SQUARES.

A.

(1.) $(2x-3y)\,(2x+3y)$; $(12x-17y)\,(12x+17y)$; $(4x^2-1)\,(4x^2+1)$.

(2.) $(2a-b-c)\,(2a-b+c)$; $(4x+y-z)\,(4x+y+z)$; $(3m+2n+p)\,(3m+2n-p)$.

(3.) $(200)\,(198)$; $(x-y+z)\,(x+y-z)$; $(a-3b+c)\,(a-b-c)$.

(4.) $(x^2+y^2+z^2+2xz)\,(x^2+y^2+z^2-2xz)$; $(a-b+x+y)\,(a-b-x-y)$.

Page 12. (5.) $(b+c-a+d)\,(b+c+a-d)\,(a+d-b+c)\,(a+d+b-c)$; $(a+b+c)\,(a+b-c)\,(c+a-b)\,(c-a+b)$.

(6.) $(x^n-y^n)\,(x^n+y^n)$; $16(1+x)\,(1-x)$; $4(a+c)\,(b+d)$.

(7.) $(x^2+y^2+z^2)\,(x^2+y^2+z^2-2xy-2xz-2yz)$.

(8.) $15(x-2y)\,(x+2y)$; $3(9x^2-4y^2)\,(9x^2+4y^2)$; $(1-2ab^n)\,(1+2ab^n)$.

(9.) $(3a-5b+4c)\,(-a+b-4c)$; $(a^2+a-b^2+b)\,(a^2+a+b^2-b)$.

(10.) $(x+2z)\,(x-2y)$.

B.

(1.) $7x-5y+z$. (2.) $(x^2+y^2)^2-z^4$.

(3.) $(a^2+b^2+c^2+d^2)\,(a^2+b^2-c^2-d^2)$.

(4.) Factor dividend. (5.) 840.

Apply difference of squares to 6, 7, 8, 9 and 10.

EXERCISE XV.

EXTENDED APPLICATION OF $(x \pm y)^2$ AND $x^2 - y^2$.

A.

(1.) $(x^2-2xy+2y^2)(x^2+2xy+2y^2)$;
$(x^2-3x-3)(x^2+3x-3)$; $(x^2-x+1)(x^2+x+1)$.

(2.) $(16x^2+4x+1)(16x^2-4x+1)$;
$(x^2+3x+7)(x^2-3x+7)$; $(a^2+3ab-b^2)(a^2-3ab-b^2)$.

(3.) $(2a^2-5ab-3b^2)(2a^2+5ab-3b^2)$; $(3x^2-xy+y^2)$
$(3x^2+xy+y^2)$; $(x^2-4xy-y^2)(x^2+4xy-y^2)$.

(4.) $(m^2+4mn-n^2)(m^2-4mn-n^2)$; (x^4-x^2+1)
(x^4+x^2+1); $(c^2-ac+a^2)(c^2+ac+a^2)$.

(5.) $(a^2-4ab+8b^2)(a^2+4ab+8b^2)$; $(25a^2-5a+1)$
$(25a^2+5a+1)$; $(a^2-5ab+3b^2)(a^2+5ab+3b^2)$.

(6.) $\left(3a^2-ab+\frac{b^2}{2}\right)\left(3a^2+ab+\frac{b^2}{2}\right)$;
$\left(2x^2-\frac{x}{2}-3\right)\left(2x^2+\frac{x}{2}-3\right)$.

Page 13. (7.) $\left(x^2-\frac{4x}{3}+\frac{16}{9}\right)\left(x^2+\frac{4x}{3}+\frac{16}{9}\right)$;
$(x^2-5x+25)(x^2+5x+25)$.

(8.) $(4a^2-3ab-b^2)(4a^2+3ab-b^2)$;
$(x^{2m}-4x^my^m+8y^{2m})(x^{2m}+4x^my^m+8y^{2m})$;
$(x^2-3x+1)(x^2+3x+1)$.

(9.) $(4m^2-2mn-3n^2)(4m^2+2mn-3n^2)$.

(10.) $\{(x+y)^2-3z(x+y)+z^2\}\{(x+y)^2+3z(x+y)$
$+z^2\}$;
$\{(a+b)^2-c(a+b)-c^2\}\{(a+b)^2+c(a+b)-c^2)\}$.

B.

(1.) $(3a^2+3ab+2b^2)(3a^2-3ab+2b^2)$;
$(x^2+x+4)(x^2-x+4)$;
$(4x^2-6xy+9y^2)(4x^2+6xy+9y^2)$.

(2.) $\left(\frac{a^2}{3}-ab+\frac{b^2}{4}\right)\left(\frac{a^2}{3}+ab+\frac{b^2}{4}\right)$;
$\left(\frac{a^2}{16}-\frac{ab}{12}+\frac{b^2}{9}\right)\left(\frac{a^2}{16}+\frac{ab}{12}+\frac{b^2}{9}\right)$.

(3.) $(a^2-b^2-c^2-2bc)(a^2-b^2-c^2+2bc)$.

(4.) $\{x^2-2x(y+z)+2(y+z)^2\}$
$\{x^2+2x(y+z)+2(y+z)^2\}$; $(a^2+3b^2)(3a^2+b^2)$.

(5.) $\left(\frac{1}{a^2}+\frac{1}{ax}+\frac{1}{x^2}\right)\left(\frac{1}{a^2}-\frac{1}{ax}+\frac{1}{x^2}\right)$;
$\left(\frac{3}{x^2}-\frac{3}{xy}+\frac{1}{y^2}\right)\left(\frac{3}{x^2}+\frac{3}{xy}+\frac{1}{y^2}\right)$.

(6.) $4(a^2+5ab-2b^2)(b^2+5ab-2a^2)$.

(7.) $\left\{4a^2-5a(b-c)+2(b-c)^2\right\}\left\{4a^2+5a(b-c)+2(b-c)^2\right\}$.

(8.) $\left\{(x^2-xy+y^2)^2-3(x^3+y^3)+(x+y)^2\right\}\left\{(x^2-xy+y^2)^2+3(x^3+y^3)+(x+y)^2\right\}$.

(9.) $(a^2-2ab+5b^2)(5a^2-2ab+b^2)$; $(2x^2-xy-3y^2)(2x^2+xy-3y^2)$.

(10.) $\left(\frac{1}{a^2}-\frac{2}{ab}+\frac{3}{b^2}\right)\left(\frac{1}{a^2}+\frac{2}{ab}+\frac{3}{b^2}\right)$;
$\left(\frac{4}{a^2}-\frac{2}{ab}+\frac{1}{b^2}\right)\left(\frac{4}{a^2}+\frac{2}{ab}+\frac{1}{b^2}\right)$.

EXERCISE XVI.

TRINOMIALS.

A.

(1.) $(x+2)(x+6)$; $(x+4)(x+5)$; $(x+37)(x+10)$.
(2.) $(x+40)(x+49)$; $(x-13)(x-14)$; $(x-25)(x+6)$.
(3.) $(x+20)(x-4)$; $(x-26)(x-62)$; $(x-40)(x+3)$.
(4.) $(5x+4)(3x+1)$; $(3x+2y)(2x-3y)$;
$(4c-7a)(4c+3a)$.
(5.) $(x+\frac{4}{5})(x-\frac{5}{4})$; $(x+\frac{2}{5})(x-\frac{5}{2})$; $(x-1)(x-\frac{17}{18})$.
(6.) $(x+12)(x+21)$; $(x-99)(x+7)$; $(x-48)(x+11)$.
(7.) $(3x-7y)(7x-2y)$; $(x^2+\frac{1}{3})(x^2-\frac{3}{4})$; $(x-\frac{3}{2})(x-\frac{4}{9})$.
(8.) $13x(13y)$. (9.) $\left\{4(x+2)^2-x^2\right\}\left\{(x+2)^2-9x^2\right\}$.
(10.) $\left\{(a-b)^m-11\right\}\left\{(a-b)^m-33\right\}$.

B.

Page 14. (1.) $(8x-9)(9x-8)$; $(4x-5)(2x-7)$.
(2.) $(3x-4y)(8x+y)$; $(5x-1)(2x-3)$.
(3.) $(15x+99)(x+1)$; $(4x-3)(3x+7)$.
(4.) $(2a+3b)(3a-5b)$; $(4z-5x)(8z+4x)$.
(5.) Multiply by 4 times co-efficient of first term thus— $4\times 413^2x^2-4\times 413\times 606xy-4\times 413\times 299y^2$, then add $(606y)^2-(606y)^2$, ∴ we have difference of two squares $=(826x-606y)^2-(928y)^2$. Factor in ordinary way and divide result by 4×413, $=(59x+23y)(7x-13y)$. Second part $(17x+8y)(12x-25y)$.

(6.) $(5x+151)(9x-149)$; $(6x+53)(5x+99)$.
(7.) $(13x-495)(6x-97)$; $(7x+165)(8x-169)$.
(8.) $(6x-111)(7x+107)$; $(8x+109)(12x-91)$.
(9.) $(17x+215)(2x-143)$. (10.) $(7a-437)(4a+191)$.

EXERCISE XVII.

POLYNOMIALS.

A.

(1.) $(4x-2y)(5x+3y-2)$. (2.) $(2a-5b)(3a+4b-3)$.
(3.) $(x-6y)(7-2x-3y)$. (4.) $(x+5y)(3x+4y+2)$.
(5.) $(x-3y)(x+2y-4z)$. (6.) $(3x-2y)(6x-4y+3z)$.
(7.) $(3x+2y-4z)(2x-3y+5z)$.
(8.) $(11a-b-6c)(5a-6b+2c)$.
(9.) $(2a-3b+4c)(3a-7b-c)$.
(10.) $(3m-n-5r)(m+n+r)$.

B.

(1.) $(7x+6y+8)(x-y-2)$.
(2.) $(5x-5y-22)(4x+y+4)$.
(3.) $(4x+5y)(5x-4y+7)$. (4.) $(x+3y)(x-4y-5)$.
(5.) $(3x-2y-2z)(2x-3y+4z)$.
Page 15. (6.) $(2a-5b-7c)(2a+3b+3c)$.
(7.) $(5x+4y-6)(3x-7y)$. (8). $(5a-4b-2)(a-3b)$.
(9.) $(6x-4y+3)(3x-5y)$. (10.) $(5x+4y-6)(4x+3y)$.

C.

(1.) $(a-3b)(4a+7b+4)$. (2.) $(3x+4y-8z)(2x-5y+6z)$.
(3.) $(2x+y+7z)(x+2y+3z)$. (4.) $(3x+5y)(8x-y+4)$.
(5.) $(3x-4y+z)(x+y+z)$.
(6.) $(2x+3m-4s)(x+m+3s)$. (7.) $4m+3p-9n$.
(8.) . (9.) .
(10.) Factor in ordinary way $(8x-3y+6z)(2x-5y+8z)$. Then $\frac{1}{2}$ the sum = one quantity, $\frac{1}{2}$ the difference = the other $(5x-4y+7z)^2-(3x+y-z)^2$.

EXERCISE XVIII.

APPLICATION OF $x^3 \pm y^3$.

(1.) $(a+b)(a^2-ab+b^2)$; $(a+x+y)(a^2+2ax+x^2-ay-xy+y^2)$; $(m+n+p+q)(m+n)^2-(m+n)(p+q)+(p+q)^2$.

(2.) $2(m^2+n^2)(m^4+5m^2n^2+n^4)$; $(a^2+b^2)(a^4-a^2b^2+b^4)$.

(3.) $(a^4+b^4)(a^8-a^4b^4+b^8)$; $(a^5+b^5)(a^{10}-a^5b^5+b^{10})$; $(2a+3b)(4a^2-6ab+9b^2)$.

(4.) $(x^6+y^3)(x^{12}-x^6y^3+y^6)$; $(5x^7+8y^8)(25x^{14}-40x^7y^8+64q^{16})$; $\{(a-b-c)\}\{a^2+ab+ac+b^2+2bc+c^2\}$.

(5.) $(2x-4y)(4x^2+8xy+16y^2)$; $(a^8-b^{11})(a^{16}+a^8b^{11}+b^{22})$; $(x-a+b)(x^2-2ax+a^2-bx+ab+b^2)$.

(6.) (7.) (9.) (10.) Use $\frac{x^3 \pm y^3}{x \pm y}$, etc.

Page 16. (8.) $(x+a)^2-b(x+a)+b^2$.

EXERCISE XIX.

GENERAL EXERCISE IN FACTORING.

A.

(1.) $(x+y)(ax+ay-bc)$; $(5p+24)(3p-1)$.

(2.) $2a(2b-2c)$; $(a-b-c-2)(a+b+c)$.

(3.) $(2x+3y+z)(x+4y+3z)$; $(x^7+4y^7)(x^7-y^7)$.

(4.) $(b-c)(x+a)^2$; $(2a+2b+1)(a+b+2)$.

(5.) $(x+y)(x-y)(x^2+xy+y^2)$; $(a-1-b)(a^2-2a+1+ab-b+b^2)$.

(6.) $(a^2+b^2)(c^2+d^2)$; $(x^2+5x+4)(x^2+5x+6)$.

(7.) $(x+y)(x^2+xy+y^2)(x^2-xy+y^2)$; $(x^2+1)(x^2+x-1)$.

(8.) $(a-1)(a^2-a+1)$;
write expression x^3+1+4x^2+5x+1 factor by parts;
$(x+1)(x^2+3x+2)$; $(x+1)(x+1)(x+3)$.

(9.) $(x+1)(x+2)(x+3)$; $(x-1)(x-2)(x-4)$: $(x-2)(x-3)(x-4)$.

(10.) $(x+1)(x-2)(x+3)$: $(x-2)(x-4)(x+5)$. $(x-1)(x+2)(x-3)$.

B.

(1.) $(x+2)(x+3)(2x+1)$; $(x+1)(x+2)(3x+2)$.
(2.) $(x+1)(3x^2+2x+5)$; $(x+1)(x+3)(2x-1)$.
(3.) $(2x+1)(2x-1)(x+2)$; $(3x+2)(3x-2)(x-5)$.
(4.) $(x+2)(x-3)(6x-5)$; $(a+2b)^3$.
(5.) . (6.) $(x-1)(x+1)(x^2-px+q)$.
(7.) $3abc$. (8.) $(x+3)(x+6)(x^2+9x-2)$.
(9.) $(x-1)(x+1)(x^2-10)$. (10.) $(x+1)(x-1)(x-2)$.

EXERCISE XX.

H. C. F.

A.

Page 17. (1.) $2(a-x)$; $a+b$. (2.) $x-a$. (3.) $x-7$.
(4.) $x-12$. (5.) x^2-2. (6.) $x-2$. (7.) $2x+3$.
(8.) $12x^2-5$. (9.) $a-1$. (10.) $a^2(3a+2)$.

B.

(1.) $x+6$. (2.) $7x-2y$. (3.) $x^2+2xy-y^2$.
(4.) $x-3$. (5.) $3x^2-2$. (6.) $x+y$.
(7.) $x+3$; $\therefore x=-3$. (8.) $a=6$. (9.) $x-1$.
(10.) $x^3(x-1)(x-2)^2$.

EXERCISE XXI.

L. C. M.

Page 18. (1.) $6x^2(3x-1)$; (2.) $(x+2)(x+3)(3x+2)$.
(3.) $(x-1)(x+1)(x+2)$.
(4.) $(x+4)(x+1)(x-2)(x+3)$.
(5.) $(a^2-1)(a^2-9)(a+5)$.
(6.) $(x+3)(x+4)(x+4)(x+5)$.
(7.) $(x-1)(x-2)(x-3)(x-4)$. (8.) $(a^2+1)(a^6-1)$.
(9.) $(x^2-y^2)^2(x^4+x^2y^2+y^4)$.
(10.) $(x-3)(x-8)(x+8)(x+9)$.

EXERCISE XXII.

GENERAL EXERCISE—H. C. F. AND L. C. M.

(1.) H. C. F. $=a-1$; L. C. M. $a^4-5a^3+7a^2-a-2$.
(2.) H. C. F. $=(x+1)(x+2)$;
L. C. M. $(x+1)(x+2)(x+3)(x-2)$.
Note—Question should be x^3+x^2, etc.
(3.) $(x^2+5x+6)(x^2+7x+8)$. (4.) $a=12$.
(5.) $a=20$. (6.) $a=12$, $b=12$. (7.) $a=10$.
(8.) $b=2$. (9.) $c=-114$. (10.) $a=9$. (11.) abx^2.
Page 19. (12.) $m=28$. (13.) $4ax$; $2bx$ and $2ax$; $4bx$.
(14.) $a=9$; other expression $x+2$. (15.) .
(16.) . (17.) . (18.) x^2+6x+9.
(19.) . (20.) .
(21.) $(x+1)(x+2)(x+3)(x-7)$, $\therefore x=7$ to make each vanish.
(22.) H. C. F. $=x-1$, $\therefore x=1$ to make each vanish.

FRACTIONS.

EXERCISE XXIII.

Page 20. (1.) $\frac{a}{ab-bc}$; $\frac{3}{(x-1)(x-2)}$. (2.) $\frac{x-b}{x+c}$; $\frac{a}{a+4b}$.

(3.) $a+b+c$; $\frac{x}{x-2}$. (4.) $\frac{x-y}{4z}$; 1. (5.) $\frac{5x}{2}$; $\frac{x}{x-3}$.

(6.) $\frac{(a-1)^2}{a^2-3a+1}$; $\frac{a-2b}{a^2+ab+b^2}$. (7.) $\frac{2}{x-3y}$.

(8.) $-\frac{2a+4}{(a-1)(a-2)}$. (9.) $\frac{1}{(x-1)(3-x)}$. (10.) $\frac{221-30x}{14}$.

EXERCISE XXIV.

(1.) $\frac{10(x^2+1)}{(x^2-1)^2}$. (2.) 3. (3.) $\frac{4x+1}{3x+1}$.

Page 21. (4.) $\frac{x-3y}{x+3y}$. (5.) $\frac{x}{(x-2a)^2}$. (6.) $\frac{x+1}{x+2}$. (7.) 0.

(8.) $4a^2-9x^2$. (9.) $\frac{2a(a^2+b^2)^2}{(a^2-b^2)^2}$. (10.) x.

EXERCISE XXV.

(1.) $\frac{ab+ac+bc}{abc}$; 0. (2.) $\frac{a^2+x^2}{a^2-x^2}$; $\frac{24xy}{9x^2-4y^2}$.

(3.) $\frac{2x^3}{x^4+x^2+1}$; 0. (4.) $\frac{2}{a-b}$. (5.) $\frac{2a^3+a^2+ab^2-b^3}{a(a^2-b^2)}$.

(6.) 0. (7.) $\frac{x-3}{x+4}$. (8.) 1. (9.) a. (10.) $\frac{3a-4b}{6a+b}$.

EXERCISE XXVI.

Page 22. (1.) 1. (2.) $\frac{b+a}{b-a}$; $\frac{(2a+3)(4a+5)}{(3a+4)(5a+6)}$. (3.) 1; x.

(4.) $\frac{8a^4}{a^4-x^4}$. (5.) 1. (6.) $\frac{x^6-y^6}{x^3y^3}$. (7.) $\frac{a^2+b^2}{2ab}$.

(8.) $-\frac{ax}{x^2+a^2}$. (9.) 1. (10.) 0.

EQUATIONS.

EXERCISE XXVII.

A.

Page 23. (1.) $x=7$. (2.) $x=15$. (3.) $x=15$. (4.) $x=\frac{a}{9}\cdot$
(5.) $x=3$. (6.) $x=8$. (7.) $x=13$. (8.) $x=28$.
(9.) $x=19$. (10.) $x=16$.

B.

(1.) $x=3$. (2.) $x=5$. (3.) $x=\frac{24}{113}\cdot$
(4.) $x=-107$. (5.) $x=7\frac{2}{3}$. (6.) $x=2\frac{3}{7}$. (7.) $x=6$.
Page 24. (8.) $x=3\frac{11}{79}\cdot$ (9.) $x=7$. (10.) $x=8$.

C.

(1.) $x=20$. (2.) $x=-2$. (3.) $x=4$. (4.) $x=8$.
(5.) $x=4$. (6.) $x=2$. (7.) $x=3$. (8.) $x=5$.
(9.) $x=\frac{7}{4}$. (10.) $x=8$.

D.

(1.) $x=2$. (2.) $x=2$. (3.) $x=7$. (4.) $x=4$.
(5.) $x=8$.
Page 25. (6.) $x=7$. (7) $x=\frac{b^2-2bc+ac}{a-c}\cdot$ (8.) $x=-2$.
(9.) $x=1$. (10.) $x=-6$.

E.

(1.) $x=4$. (2.) $x=-2\frac{1}{2}$. (3.) $x=\frac{cd-ab}{a+b-c-d}\cdot$
(4.) $x=\frac{ac}{d-b}$; $x=\frac{a^2-b^2}{2b}\cdot$ (5.) $x=abc$.
(6.) $x=\frac{1}{ab}$; $x=\frac{ab}{a-b}\cdot$ (7.) $x=-b$. (8.) $x=\frac{m+n}{2}\cdot$
(9.) $x=4a$. (10.) $x=\frac{m}{2}\cdot$

F.

(1.) $x=11$. (2.) $x=-21$. (3.) $x=24$. (4.) $x=5$.
Page 26. (5.) $x=7$. (6.) $x=13$. (7.) $x=10$. (8.) $x=8$.
(9.) $x=\frac{1}{5}$. (10.) $x=-2$.

EXERCISE XXVIII.

PROBLEMS.

(1.) 22 miles. (2.) \$180 for horse, \$100 for buggy.
(3.) A \$93.50, B \$280.50, C \$1122. (4.) $4\frac{1}{2}$ hours.
(5.) \$3200. (6.) 240 yds. long, 80 yds. wide.
(7.) A \$142.50, B \$47.50. (8.) 15 at \$38 and 8 at \$50.
(9.) \$15.00.
Page 27. (10.) \$9. (11.) \$800, \$3200, \$1000. (12.) 50 yds.
(13.) \$1.60. (14.) \$20000. (15.) \$750.
(16.) 90 head. (17.) 18, 22, 10, 40.
(18.) 40 and 35 bags. (19.) A \$2542, B \$2422, C \$2436.
(20.) 68. (21.) \$11100. (22.) 182 and 10.
(23.) A \$648, B \$472, C \$416.
Page 28. (24.) \$18000. (25.) \$2400, \$1000. (26.) 11 horses.
(27.) $4\frac{13}{15}$ lbs. (28.) 69 and 81. (29.) 144 sq. yds.
(30.) \$1050. (31.) 84. (32.) 18x12 ft.
(33.) 16 and 24. (34.) A \$70, B \$120, C \$190.
(35.) 10 vols. (36.) \$650, \$750, \$650, \$450.
(37.) 24000 men.

EXERCISE XXIX.

Page 29. (1.) $x=7$; $y=2$. (2.) $x=7$; $y=3$.
(3.) $x=7$; $y=3$. (4.) $x=3$; $y=5$.
(5.) $x=5$; $y=1$. (6) $x=96$; $y=72$.
(7.) $x=15$; $y=24$. (8.) $x=12$; $y=12$.
(9.) $x=\frac{1}{2}$; $y=\frac{1}{9}$. (10.) $x=18$; $y=10$.
Note.—Question should be $\frac{x}{3}+\frac{y}{5}=8$.
(11.) $x=60$; $y=36$. (12.) $x=12$; $y=8$.
(13.) $x=a+b$; $y=a-b$. (14.) $x=\frac{n-m}{a-1}$; $y=\frac{am-n}{a-1}$.
(15.) $x=\frac{m^2-n^2}{am-bn}$; $y=\frac{n^2-m^2}{an-bm}$.
(16.) $x=\frac{bc}{a+b}$; $y=\frac{ac}{a+b}$. (17.) $x=\frac{1}{2}$; $y=2$.
(18.) $x=2$; $y=1$. (19.) $x=2$; $y=4$; $z=6$.
(20.) $x=-1$; $y=1$; $z=0$. (21.) $x=3$; $y=\frac{1}{3}$; $z=-1$.
(22.) $x=1$; $y=2$; $z=3$. (23.) $x=3$; $y=5$; $z=4$.

Page 30. (24.) $x=10$; $y=6$; $z=0$. (25.) $x=\frac{1}{2}$; $y=\frac{1}{3}$; $z=\frac{1}{5}$.
(26.) $x=\frac{1}{2}$; $y=\frac{1}{4}$; $z=\frac{1}{6}$.
(27.) $x=2$; $y=1$; $z=4$; $p=3$.
(28.) $x=16$; $y=7{\cdot}75$; $z=5{\cdot}5$.
(29.) $x=\frac{a}{2}$; $y=\frac{b}{3}$; $z=\frac{c}{4}$.
(30.) $x=\frac{c+a}{2}$; $y=\frac{b-c}{2}$; $z=\frac{a-b}{2}$.

EXERCISE XXX.

(1.) 17 yds.; 13 yds. (2.) $\frac{2}{3}$ ton ; $1\frac{1}{2}$ tons. (3.) 45.
(4.) 60c.; 16c. (5.) \$1.00 ; 60c. (6.) 16 ; 25.
(7.) A 100c., B 60c.
Page 31. (8.) \$48 = cow, \$96 = horse. (9.) 8 and 15.
(10.) \$180 and \$120. (11.) 3, 4, 5. (12.) 8, 12, 18.
(13.) A = \$1.00, B = \$1.12. (14.) 31 and 23.
(15.) John = \$22, Tom = \$26. (16.) 41 and 7.
(17.) 72c. and 40c. (18.) 35 and 65. (19.) 24.
(20.) $\frac{5}{6}$. (21.) 40 and 90. (22.) $\frac{3}{15}$.
Page 32. (23.) 40 and 65. (24.) A = \$232, B = \$332.
(25.) A = \$31, B = \$27. (26.) $\frac{6}{18}$. (27.) $\frac{35}{41}$.
(28.) $\frac{2}{3}$ and $\frac{5}{6}$. (29.) 26. (30.) 75. (31.) 69.
(32.) $7\frac{1}{2}$ and $4\frac{1}{2}$ hours. (33.) \$5000 each. (34.) 35.
(35.) 72, 64, 56, 48.
Page 33. (36.) \$540 and \$360.
(37.) 30 and 50, and 70 and 20, *or* 60 and 20, and 40 and 50.
(38.) 12 sheep, \$40. (39.) 10, 22, 26. (40.) $2\frac{1}{2}$s., 2s.
(41.) 3, 5, 8. (42.) \$360 and \$600.
(43.) 80 and 120. (44.) 48c. and 40c.

MISCELLANEOUS EXERCISES.

A.

Page 34. (1.) $x^2-4x+11$. (2.) 0. (3.) x^2-6x+8.
(4.) $(x-1)(x-3)$. (5.) $16xy$. (6.) $-4ab$.
(7.) $2x$. (8.) $(a+b)(a-2)$. (9.) x^6+8x^3-153.
(10.) m^3+n^3. (11.) 43. (12.) $x^2+8xy-7y^2$.
(13.) $ab(a+2b)(2a+b)$.
(14.) $x(3x+2y)$; $5b(3a^2-2b+3c)$. (15.) $x-y$.

(16.) -47. (17.) $a^2-b^2-\frac{2a}{c}+\frac{1}{c^2}\cdot$ (18.) $abc(4a^3-3b+2c^3)$.

(19.) \$70. (20.) $m^3-n^3-3mn(m-n)$. (21.) $\frac{a^2}{2x}\cdot$

(22.) $(8x+23)(x-2)$. (23.) $10-8x$. (24.) $x=5$.

Page 35. (25.) $(x+4y+5)(x+y)$. (26.) 0. (27.) $12a-14b$.

(28.) $(4x-11)(7x-8)$. (29.) $x=7$. (30.) 60 and 40.

(31.) $28a$. (32.) -2. (33.) 94.

(34.) $(ax-ay)(ax+ay)$. (35.) $\frac{x^2-4}{x^2-9}\cdot$ (36.) $x-2$.

(37.) 4. (38.) $\frac{5x+1}{x^4-1}\cdot$ (39.) $\frac{a+b}{a-b}\cdot$ (40.) $(x+2)(x+3)$.

(41.) 1. (42.) x^2+3x+5. (43.) 4.

(44.) \$150 and \$120. (45.) $(x-1)(x-4)(x+1)(x+4)$.

(46.) $\frac{3x+y}{3x-y}\cdot$ (47.) $x^3+y^3+z^3-3xyz$.

(48.) $(2x-3y+z)^2$.

Page 36. (49.) 9. (50.) $a(a^2+b^2)$. (51.) 0. (52.) $x=4$.

(53.) a^8-1. (54.) $x=7$. (55.) a^2b-3. (56.) 74.

(57.) $-2y$. (58.) $x=7$. (59.) Apply $\frac{x^2-y^2}{x-y}$, $56(x+y)$.

(60.) $\frac{x^2+2x+3}{x^2-2x+3}\cdot$ (61.) $a=65$.

(62.) $(4x-15y)(6x+5y)$. (63.) $2(m^2+q^2)(x^2+y^2)$.

(64.) 14. (65.) $x=7$. (66.) -1. (67.) $a^{10}-1$.

(68.) $x=7$. (69.) $x=3$. (70.) $(x-4)(x-5)(x+11)$.

(71.) A = \$160, B = \$400. (72.) $(2x-11)(x-5)$.

Page 37. (73.) $x=10\frac{1}{4}$. (74.) $(m-n+k-l)(m-n-k+l)$.

(75.) $(a-b)(a+m+l)$.

B.

(1.) 46. (2.) $\frac{2x^3}{x^4+x^2+1}\cdot$

(3.) $x^3+(a+b-c)x^2+(ab-ac-bc)x-abc$.

(4.) $x^3+10x^2-47x-504$. (5.) $a^{12}+a^9+a^6+a^3+1$.

(6.) $\frac{3x-4y}{60}\cdot$ (7.) $x=2\frac{2}{3}$. (8.) Each \$10. (9.) $\frac{x^2}{a+x}\cdot$

(10.) $x=7$. (11.) $-2bc$. (12.) $x=3$. (13.) $\frac{2}{1-y}\cdot$

(14.) 72. (15.) 4. (16.) 0.

(17.) Examine for complete square a^2.

(18.) $a^2+b^2+c^2-2bc+ab-ac$. (19.) $2xy(x^2+y^2)$.

Page 38. (20.) 1 of first, 3 of second. (21.) 0. (22.) $\frac{4(a^2+b^2)}{(a-b)^2}\cdot$

(23.) $x+3$. (24.) $x=8$. (25.) $\frac{1}{5}$.

(26.) $16y^3-27z^3-36yz(4y-3z)$. (27.) xy.

(28.) Multiply first equation by a, second by b, third by c, etc.

(29.) $\frac{1}{6}$. (30.) $x=\frac{1}{2}$. (31.) 0. (32.) $\frac{x-3}{2x^2+5}$.

(33.) $(m+q+n-p)(m+q-n+p)$. (34.) $x+\frac{1}{x}$.

(35.) $(x+y+a)(x+y+b)$. (36.) $3+6b$.

(37.) $x^2-5ax+7a^2$.

(38.) $m=-510$. (39.) $x=10a$. (40.) $\frac{2x}{1+x^2+x^4}$.

(41.) $\frac{x^2+x-12}{x^2-x-12}$. (42.) n.

Page 39. (43.) Second, x^2-2x+2. (44.) $\frac{x+7}{x+5}$. (45.) .

(46.) . (47.) -30. (48.) $1-a^8b^8$.

(49.) x^2-xy+y^2, (ii) $(x^4-x^2y^2+y^4)(x^2-xy+y^2)(x^2+xy+y^2)$.

(50.) 39. (51.) $\frac{1-y}{x}$. (52.) x^4-4x^2+16. (53.) -8.

(54.) $\frac{a+c}{a-c}$. (55.) $\frac{x+a}{2a}$. (56.) 8. (57.) $\frac{a^2-4b^2}{a}$.

(58.) $\frac{a^3}{b^3}+\frac{b^3}{a^3}+3\left(\frac{a}{b}+\frac{b}{a}\right)$. (59.) $\frac{8x^3}{a^3}-\frac{27y^3}{b^3}$.

(60.) $\frac{2}{(1-4a^2)(1+a)}$.

(61.) $a(a-1)(a+1)(a+3)(a-6)$. (62.) $\frac{2a}{c^2}$.

(63.) C. (64.) $\frac{3x+2}{5x+1}$. (65.) -1.

Page 40. (66.) $(x-2y+z)(x^2+4y^2+z^2+2xy+2yz-xz)$.

(67.) $(x+2y+z)(x^2+4y^2+z^2-2yz-xz-2xy)$.

(68.) $(2a+3b-c)(4a^2+9b^2+c^2+3bc+2ac-6ab)$.

(69.) $(2a-3b-c)(4a^2+9b^2+c^2-3bc+2ac+6ab)$.

(70.) $(x+2y-1)(x^2+4y^2-2xy+x+2y+1)$.

(71.) $(x-2y-1)(x^2+4y^2+x+2xy-2y+1)$.

(72.) $(x+1)(x+1)(x-2)(x-3)$.

(73.) Hint.—If there is a binomial factor the co-efficient of x is unity, and the second term must be ± one of the factors of 105. Using the remainder theorem, the expression vanishes when $1, 3, -5$ or -7 is put for x, $\therefore$ factors $=(x-1)(x-3)(x+5)(x+7)$.

(74.) Write $(x^2-7x)^2+22(x^2-7x)+120$, etc., $(x-2)(x-3)(x-4)(x-5)$.

(75.) $(x-3)(x-5)(x+4)(x+8)$. (76.) .

(77.) $a^2+b^2-c^2-d^2$.

(78.) Write expression $(a^2-bc)^3-(a^2-bc)(b^2-ac)(c^2-ab)$ or $(a^2-bc)\{(a^2-bc)^2-(b^2-ac)(c^2-ab)\}+(b^2-ac)\{(b^2-ac)^2-(c^2-ab)(a^2-bc)\}+(c^2-ab)\{(c^2-ab)^2-(a^2-bc)(b^2-ac)\}=a(a^2-bc)\{(a^3+b^3+c^3-3abc)+b(b^2-ac)(a^3+b^3+c^3-3abc)+c(c^2-ab)\}$, etc.

(79.) Write expression $x^4+10x^3+25x^2-8(x^2+5x)-33=(x^2+5x-11)(x^2+5x+3)$. (80.) .

(81.) $(a^2-b^2)(a^2-4b^2)$. (82.) m.

(83.) $(2a-x)^2(a-x)^2$. (84.) $x=-\frac{110}{219}$.

(85.) Note.– Question should be x^2+6mx, etc., $\therefore$ factors $(x+5m+3n)(x+m-3n)$.

(86.) $x^8+\frac{x^4}{y^2}+\frac{1}{y^4}$. (87.) $9(a+b)(a^2+ab+b^2)$.

(88.) . (89.) x^2+a^2. (90.) $R=61$.

EXERCISE I.

EXAMINATION PAPERS.

Page 41. (1.) 96. (2.) $\frac{1}{2}$. (3.) $4\frac{1}{2}$. (4.) 12. (5.) 21.

(6.) $3x^3-2x^2-5x-2$. (7.) $(m-n)^2-(p-q)^2$.

(8.) $a^8+a^4b^4+b^8$.

(9.) $(a-3b)(a+2b)$; $(a^2+b^2-5ab)(a^2+b^2+5ab)$; $(5x+4y)(3x+4y)$.

(10.) Apply principle, difference of square.

EXERCISE II.

(1.) $\frac{4x}{x^2-1}$. (2.) $(1+a-b+c)(1-a+b+c)$.

(3.) $x^2+y^2+z^2-xy-xz-yz$. (4.) .

(5.) $\frac{1}{x-y}$. (6.) $(ac+bd)^2-(ad+bc)^2$.

(7.) $(3x+2y)(2x-3y)$; $(x^4+y^4)(x^8-x^4y^4+y^8)$; $(2a+3b+1)(a+2b+2)$. (8.) -1. (9.) $\frac{2x+3}{2x-3}$.

(10.) $(a-b)(b-c)(c-a)$.

EXERCISE III.

Page 42. (1.) $1\frac{1}{2}$.

(2.) Reduce each fraction to a mixed number, hence $x=2$. (3.) A = \$840, B = \$600, C = \$840.

(4.) Put in form of fraction and cancel, $= x^2 - xy + y^2$.

(5.) $2x^2 - 3xy + 7y^2$. (6.) $x^2 + 3x + 1$.

(7.) $4(x^2 - xy + y^2)\ (x^2 + xy + y^2)$; $(b - c + a)\ (b - c - a)$; $(2a - x)\ (a + 2x)$.

(8.) $x = 9$. (9.) $x = 111$. (10.) .

EXERCISE IV.

(1.) x. (2.) $x^2(x-1)(x+2)(x+3)$. (3.) $c = 1$, (ii) No.

(4.) . (5.) $x^2 + 1 + \frac{1}{x^2}$.

(6.) Factor denominator and cancel, 1.

(7.) Apply $\frac{x^3+y^3}{x+y}$, etc., $2a^4 + 10a^2b^2 + 2b^4$. (8.) $x = 20$.

(9.) $x = 7$. (10.) Factor dividend, $(x + y - z)^2$.

EXERCISE V.

Page 43. (1.) Apply $\frac{x^3+y^3}{x+y}$, etc. (2.) $-2b$. (3.) 74. (4.) 13.

(5.) $(a+b)^4 - c^4$.

(6.) Apply $\frac{x^3+y^3}{x+y}$, etc., $(x+a)^2 - b(x+a) + b^2$.

(7.) -14. (8.) $(x-1)\ (x+2)\ (x-3)$.

(9.) $4(x^2 + y^2 + z^2)$. (10.) 8.

EXERCISE VI.

(1.) $a = -4$. (2.) $x^5 + 4x^3 + 16x$.

(3.) Write expression $(x^2 - x)^3 - (2)^3$, apply $\frac{x^3 - y^3}{x - y}$, etc., $(x^2 - x - 2)\ (x^4 - 2x^3 + 3x^2 - 2x + 4)$.

(4.) $(9x^2 - 5)\ (4x + 3)$. (5.) $x^3 + 24xy(x + 8y) + 512y^3$.

(6.) $x = \frac{a+b+c}{ab+ac+bc}$. (7.) $2(a+b+c)$.

(8.) 8 first-class.

EXERCISE VII.

Page 44. (1.) 4. (2.) $(a+b)\ (a-c)$.

(3.) Apply principle difference of squares, $7x + y + z$.

(4.) $2(a-b)$. (5.) $a = 4$. (6.) 1. (7.) $x - 5$.

(8.) Reduce fractions to mixed numbers, $x = 3$.

(9.) 15 and 20. (10.) $x = 7$.

EXERCISE VIII.

(1.) $x=5$.

(2.) $z(y-x)\ (y+x-z)$, $(x-a)\left(x-\frac{1}{a}\right)$; $(x-4a-4b)$ $(x+a+b)$.

(3.) 6. (4.) $60(x-1)\ (x+1)\ (x-2)$.

(5.) $\frac{(x-2)\ (x+5)}{(x-4)\ (x+3)}\cdot$ (6.) $\frac{x}{(x-2a)^2}\cdot$ (7.) $\frac{x(x+a)}{2}\cdot$

(8.) Multiply and $a+1=0$, $\therefore\ a=-1$, $b+a+1=0$, $\therefore$ $b=0$.

(9.) Apply $\frac{x^2-y^2}{x+y}$, etc. (10.) $=\left(\frac{1}{x}\right)^2-\left(\frac{1}{z}\right)^2\cdot$

EXERCISE IX.

Page 45. (1.) $x^4+x^2+1+\frac{1}{x^2}+\frac{1}{x^4}\cdot$ (2.) a^6-b^6. (3.)

(4.) $x=17$. (5.) Apply $\frac{x^3+y^3}{x+y}$, etc., $x^4-4x^2yz+7y^2z^2$.

(6.) $5\frac{5}{11}'$ and $38\frac{2}{11}'$ past 4. (8.) $\frac{(a-b)^2}{a+b}\cdot$

(9.) $x^8-x^6+2x^2-2$. (10.) $x^3+(a+b+c)x^2+(ab+ac+bc)x+abc$, $x^3-(m+n+p)x^2+(mn+mp+np)x-mnp$.

EXERCISE X.

(1.) $-(4ab+4ac)$. (2.) $(x+1)^2$.

(3.) $x^2-x+1+\frac{1}{x}+\frac{1}{x^2}\cdot$ (4.) Barley 50c., wheat 65c.

(5.) $x-4$. (6.) a^3+ab+b^2. (7.) $x=\frac{ac(b-d)}{a-c}\cdot$

(8.) $x=\frac{abc}{ab-ac+bc}\cdot$ (9.) $m-\frac{1}{n}\cdot$ (10.) $x=2$.

EXERCISE XI.

Page 46. (1.) $a+x$.

(2.) Write the expression $\frac{1-x^n}{1-x}\times\frac{1+x^n}{1+x}=\frac{1-x^{2n}}{1-x^2}=$ $1+x^2+x^4$, etc.

(3.) $x=95$. (4.) $a+b+c$. (5.) $x=-6$, $y=11$, $z=-6$.

(6.) $(x+m)\ (x+n)$. (7.) $\frac{x^3y^3}{27}+27$. (8.) $\frac{x+1}{x+2}\cdot$

(9.) $\frac{a}{a+ab+1}\cdot$ (10.)

EXERCISE XII.

(1.) Factors are $(x+1)^2(x-2)$, $\therefore$ other factor $x-2$.

(2.) . (3.) . (4.) $=400$ gallons.

(5.) $\frac{a^2}{2x}$. (6.) $2x^2-7x-3$.

(7.) $(x+2+2a-y)(x+2-2a+y)$. (8.) $y=1$.

(9.) \$752.

(10.) Write expression $x^4+54x^2+729+6x(x^2+27)-27x^2=(x^2+27)^2+6x(x^2+27)+9x^2-36x^2=(x^2+27+3x)^2-(6x)^2=(x^2+9x+27)(x^2-3x+27)$.

EXERCISE XIII.

Page 47. (1.) $4(2x^2-1)(2x^2-3x-1)$. (2.) . (3.) $x=3$.

(4.) $x(3x+4)(x-6)$. (5.) 64 miles. (6.) $x^{10}-1$.

(7.) $(6x+1)(3x+2)(3x+4)(2x-1)$.

(8.) $a^4-5a^2bc+13b^2c^2$.

(9.) $\frac{4x^2-15x+13}{x^3-6x^2+11x-6}$. (10.) $\frac{a+bx}{b+ax}$.

EXERCISE XIV.

(1.) $2x^2-5x+1$. (2.) $9(a+2x)(a-2x)(2a-x)^2$.

(3.) 20 cattle. (4.) $(x-a)(x-b)$. (5.) $(y^2+y)(x-y)$.

(6.) $\frac{1}{2}$. (7.) Apply $\frac{x^3-y^3}{x-y}$, etc.; $a^2+a(2b-3c)+(2b-3c)^2$.

(8.) $\frac{2+3x}{1+5x}$. (9.) $1+y^2+z^2-y-z-yz$. (10.) $3\frac{31}{57}$.

EXERCISE XV.

Page 48. (1.) $\frac{5}{6}$. (2.) $(x+1)(x-1)(x+a+1)(x+a-1)$.

(3.) $(a+b)(a^2+ab+b^2)$. (4.) . (5.) $\frac{m-y}{4z}$.

(6.) The denominator $=(ac+bd)^2+(ad-bc)^2$, which is greater than the numerator, etc.

(7.) $y^4+2y^3+3y^2+2y+1$. (8.) $(x+p)(x+m+n)$.

(9.) 14, 17, 20. (10.) Apply difference of squares, etc.

EXERCISE XVI.

(1.) . (2.) 10. (3.) 90 lbs. tea, 120 lbs. coffee.

(4.) Expression $a^2 - b^2 + \frac{1}{a^2} - \frac{1}{b^2} = \frac{(a^2-b^2)(a^2b^2)}{a^2b^2} + \frac{b^2-a^2}{a^2b^2}$
$= \left(\frac{a^2-b^2}{ab}\right)\left(\frac{a^2b^2}{ab}\right) = \left(\frac{a}{b} - \frac{b}{a}\right)\left(ab - \frac{1}{ab}\right)$.

(5.) $\frac{a+b}{a^2-b}$. (6.) Factor last expression, etc.

(7.) $(x+1)(x-1)(y+1)(y-1)$, $(a-b)(a-c)(b-c)$.

(8.) $x=1$. (9.) $\frac{a+b+c+1}{a-b-c}$. (10.) 0.

EXERCISE XVII.

Page 49. (1.) $x=4$. (2.) $(a^2+b^2)(c^2+d^2)=1$, etc. (3.) 0.

(4.) $x^4 - x^2yz + 7y^2z^2$.

(5.) Divide, and remainder equals zero, $\therefore a=7$, $b=16$.

(6.) $12x^2+12$.

(7.) Left side $= a^2+a^2+b^2+b^2+c^2+c^2-2ab-2ac-2bc = (a^2-2ab+b^2)+(a^2-2ac+c^2)+(b^2-2bc+c^2) = (a-b)^2+(a-c)^2-(b-c)^2$, etc.

(8.) 5 and 6. (9.) Factor as $a^3+b^3+c^3-3abc$, etc.

(10.) .

EXERCISE XVIII.

(1.) Add $=^{ns}$ and $x+y+z=0$, $\therefore (x+y+z)^2=0$, etc.

(2.) 0. (3.) $\frac{16x^{15}}{1-x^{16}}$. (4.) $\frac{x}{x+a}$. (5.) .

(6.) . (7.) . (8.) 0.

(9.) 1328 yards and 432 yards. (10.) .

EXERCISE XIX.

Page 50. (1.) $(3x-2)(9x-11)$. (2.) . (3.) 280.

(4.) $(a^2-x^2)(b^2-y^2)$. (5.) $\left(\frac{4a}{5} - \frac{3b}{2}\right)\left(\frac{16a^2}{25} + \frac{6ab}{5} + \frac{9b^2}{4}\right)$.

(6.) x^2-1. (7.) $x=1$. (8.) $x=5$.

(9.) Let m be added to each a, b, c, $\therefore (a+m)^2-(b+m)(c+m)+(b+m)^2-(c+m)(a+m)+(c+m)^2-(a+m)(b+m)$. Multiply out and add, etc.

(10.) Write $\frac{(2x)^5+(3)^5}{2x+3}$, etc., $16x^4-24x^3+36x^2-54x+81$.

EXERCISE XX.

(1.) 140 lbs., 60 lbs. (2.) $196x^2-289$. (3.) 1.
(4.) $(3x-2y)(3x+2y)(2x-3y)(2x+3y)$.
(5.) 993 yds. nearly. (6.) $-11x^2+17x-12$.
(7.) $a=7$. (8.) 4. (9.) $(2a-5b+6c)(3a+4b-8c)$.
(10.) $b=46$.

EXERCISE XXI.

Page 51. (1.) -66.
(2.) Square each $=n$ and add, $\therefore 2(x^2+y^2+z^2+xy+xz+yz)=a^2+b^2+c^2=0$, hence, etc.
(3.) a^2+3a+2. (4.) $(x+2y)(x-2y)$.
(5.) $a=13$, $k=1$. (6.) $x-1$, $x-2$, $x+3$.
(7.) $a=1-\frac{1}{b}$, $\therefore c+\dfrac{1}{1-\frac{1}{b}}=1$, hence etc.
(8.) $23\frac{1}{3}$. (9.) $(3x^2-x+7)^2$.
(10.) If $a>b$, then $a-b>0$, $\therefore a^2-b^2>2ab$ or $\frac{a}{b}+\frac{b}{a}>2$, etc.

EXERCISE XXII.

(1.) Equal. (2) $3x(x-7)$.
(3.) Apply difference of squares. (4.) $\left(\frac{a}{x}-\frac{x}{a}\right)^3$.
(5.) $x=\frac{c}{2}$.
(6.) $(7x-101)(8x+97)$; $(3x+49)(9x-83)$. (7.) 10.
Page 52. (8.) Write $\frac{m}{m-q}-\frac{q}{m+q}$, etc.
(9.) A \$60, B \$140, C \$200. (10.) $2x+3$.

EXERCISE XXIII.

(1.) Square each, add and factor. (2.) 4560.
(3.) $x=7a$. (4.) $x=\frac{a}{2b}$. (5.) \$1480. (6.) .
(7.) Write $x^3-1+8x^2-79x+70+1=(x-1)(x-5)(x+14)$. (8.) The latter.
(9.) Factor expression, and one factor is equal to zero, etc.

(10.) Transpose, and $x^2-2xy+y^2+y^2-2yz+z^2+z^2-2uz+u^2=(x-y)^2+(y-z)^2+(z+u)^2$. Since the square of any quantity is positive, $\therefore$ each expression is positive, and cannot be zero unless each quantity is zero, $\therefore x-y=0,\ y-z=0,\ z-u=0$, $\therefore x=y=z=u$.

EXERCISE XXIV.

(1.) $a^3-8a^2+23a-26$. (2.) . (3.) . (4.) .

Page 53. (5.) 29 miles. (6.) $\frac{1}{x}$. (7.) . (8.) .

(9.) $x^2-(a+b)x+ab=x^2+x+1$. Since co-efficients of like powers are equal, $\therefore a+b=-1$ and $ab=1$, $\therefore a^3+b^3=2$.

(10.) $a+b=-c$, multiply by $a-b$, etc.

EXERCISE XXV.

(1.) . (2.) $x=6$. (3.) $\frac{1}{abc}$. (4.) $x=\frac{13a}{2}$.

(5.) $a=8$. (6.) $\frac{x}{3-x}$. (7.) $5x^2-2x-1$. (8.) $x=10$.

(9.) Let $x-3$, $x-1$, $x+1$, $x+3$, be the numbers, etc.

(10.) .

EXERCISE XXVI.

Page 54. (1.) $x=10a$. (2.) Let $x=$ one, $x+d$ the other, etc.

(3.) $\frac{b^2+c^2-a^2}{2bc}+1=m+1$, etc. (4.) 216.

(5.) $a^3+2a^2b-ab^2-2b^3$. (6.) $m^2-12m+35$.

(7.) $(x^2-3x+17)(x^2+3x+17)$.

(8.) x^2-3x+2 is a factor and $=0$, $\therefore$ expression $=0$.

(9.) Write $\left(1-\frac{1}{a}+1-\frac{1}{b}+1-\frac{1}{c}-1\right)\div 2-\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$, etc., $=1$. (10.) .

EXERCISE XXVII.

(1.) Factor the expression. (2.) $\frac{4x^2+2x-1}{3x+1}$.

(3.) . (4.) . (5.) $\frac{1}{2}a+\frac{3}{2}b-\frac{1}{2}c$.

(6.) . (7.) $\frac{13}{19}$.

Page 55. (8.) 6. (9.) 405 yards. (10.) $x=a+b+c$.

EXERCISE XXVIII.

(1.) 3. (2.) $20x-32y$. (3.) $\left(x-\frac{a}{b}\right)\left(x-\frac{b}{a}\right)$.

(4.) $\frac{5}{16}$. (5.) $1+3x+6x^2+7x^3+6x^4+3x^5+x^6$.

(6.) $1+x+\frac{11}{12}x^2+\frac{1}{3}x^3+\frac{1}{9}x^4$. (7.) 6 ft.

(8.) $1-\frac{1}{4}a^2+\frac{2}{3}b+\frac{1}{9}b^2$. (9.) . (10.) $7a-5b$.

EXERCISE XXIX.

(1.) $x=a+b+c$. (2.) $2(a^4+a^2b^2+b^4)$. (3.) 291.

(4.) 1. (5.) $1-\frac{1}{6}x-\frac{1}{3}x^2+\frac{13}{36}x^3-\frac{1}{6}x^4$. (6.) $700.

Page 56. (7.) $\frac{a^3}{b^3}-\frac{b^3}{c^3}-3\frac{a}{c}\left(\frac{a}{b}-\frac{b}{c}\right)$.

(8.) Let x, $x+1$, $x+2$, $x+3$ be the numbers, etc.

(9.) Apply difference of squares. (10.) 0.

EXERCISE XXX.

(1.) $26x^2-48xy+26y^2$. (2.) . (3.) $l-m-n$.

(4.) . (5.) $4(x+a)(y+z)$.

(6.) $(4x^2+5bx)(5x+3a)$. (7.) x.

(8.) $3-4x+7x^2-10x^3$. (9.) -2.

(10.) $(7x+6y-9)(x-y+4)$.

EXERCISE XXXI.

(1.) -20. (2.) $(1+x+x^2)^3-(1-x+x^2)^3-6x(x^4+x^2+1)$ is a cube, etc.

(3.) . (4.) $\frac{b^2-m^2}{2(m-a)}$. (5.) 20 years.

Page 57. (6.) $(x+2y+3z)^2+(4y+5z)^2+(6z)^2$.

(7.) $(x-a)(x^2-b^2)$. (8.) $(x^2+6x+11)(x^2+6x+3)$.

(9.) $(x^2-6x+4)(x^2+6x+4)$.

(10.) Apply difference of squares.

EXERCISE XXXII.

(1.) 200 lbs. (2.) Apply difference of squares.

(3.) $3x^2-7x+11$.

(4.) Divide the expression by product of $(2x-3)(3x+1)$, and the co-efficients of like powers in the remainder must be equal ; $m=6$, $n=-37$.

(5.) 216. (6.) $(8a+15b)(14a-9b)$.
(7.) Factor left hand side, etc. (8.) $-(x+z)$.
(9.) $a^4-pa^3+qa^2-ra+s=0$. (10.) $\frac{4m}{4+m^2}$.

EXERCISE XXXIII.

(1.) $p=25$, $q=-24$. (2.) Apply $\frac{x^3-y^3}{x-y}$, etc.
(3.) . (4.) . (5.) .
Page 58. (6.) . (7.) $p=45$.
(8.) Write $(x)^{16}-(2)^{16}$. Hence $(x^8+2^8)(x^4+2^4)(x^2+2^2)(x+2)(x-2)$.
(9.) Write $9x^3+36x^2+12x^2+48x+4x+16=9x^2(x+4)+12x(x+4)+4(x+4)=(3x+2)^2(x+4)$.
(10.) $(2x+3y+z)(x+4y+3z)$.

EXERCISE XXXIV.

(1.) .
(2.) $(a_1+a_2+a_3-a_4)x+(a_1+a_2-a_3+a_4)y+(a_1-a_2+a_3+a_4)z$.
(3.) $a^3-3ab^2+2c^3=0$. (4.) $a=2$. (5.) .
(6.) A cube. (7.) $a=31$. (8.) -1. (9.) $x=4$.
(10.) Multiply first $=n$ by a, second by b, third by c.

EXERCISE XXXV.

(1.) -1024. (2.) . (3.) .
(4.) $(2y-x+a)(y-2x-a)$. (5.) $\frac{3x+2}{2x^2-4x-48}$.
(6.) $2\left(\frac{x}{a}+\frac{a}{x}\right)\left(\frac{m}{n}+\frac{n}{m}\right)$.
Page 59. (7.) Let $x^2+mx+\sqrt{s}$ = sq. root of expression. Square and equate co-efficients, $2m=p$ $2m\sqrt{s}=r$, ∴ $p^2s=r^2$.
(8.) $\left(\frac{x^2}{a^2}+\frac{a^2}{x^2}+\frac{x}{a}+\frac{a}{x}\right)^2$. (9.) .
(10.) Multiply each term by x^2-y^2, etc., $\frac{x^2+y^2}{2xy}$.

EXERCISE XXXVI.

(1.) $x=10$.
(2.) Factor, simplify; square remainder $=a^2+b^2+c^2-2ab+2ac-2bc$.

(3.) Factor and other factor required $=x-2$.

(4.) Expression $=\frac{(a+b)^{n-n+1}}{(a-b)^{n-n+1}}=\frac{a+b}{a-b}\cdot$

(5.) Factors are $(x-1)^2\,(x+1)$, $\therefore$ we must multiply by $(x-1)\,(x+1)^2$. (6.) $2\frac{1}{6}$ and 76.

(7.) $\frac{a}{2}-\frac{b}{3}\cdot$ (8.) .

(9.) Write expression $x^3-(x+3z)^3+29z^3+9xz(x+3z)$, etc., $=2z^3=250$. (10.) $2(a-b)$.

EXERCISE XXXVII.

(1.) Since $a-b=1$, $\therefore (a-b)^2=1$, $\therefore (a-b)^2(a+b)^2=a^2+2ab+b^2=\frac{a^3-b^3}{a-b}+ab=a^3-b^3+ab$.

(2.) Multiply by $a-1$, etc. (3.) $m=2$.

(4.) $x+y=z$, multiply by $x-y$, etc. .

Page 60. (5.) .

(6.) Since x^3+y^3 is divisible by $x+y$, $\therefore (x^{\frac{1}{3}})^3+(y^{\frac{1}{3}})^3$ is divisible by $x^{\frac{1}{3}}+y^{\frac{1}{3}}$. Result, $x^{\frac{2}{3}}-x^{\frac{1}{3}}y^{\frac{1}{3}}+y^{\frac{2}{3}}$.

(7.) $x^2+x^2+2x+1-2x^2-2x>0$, $\therefore$ expression is positive.

(8.) The former.

(9.) $x(x+y+z)\,(x^2+y^2+z^2-xy-xz-yz)$.

(10.) 35 miles.

EXERCISE XXXVIII.

(1.) $ac=b^2$; add $2bc+c^2$ to each and divide by c, etc.

(2.) If $a-b$ is positive, $\therefore a>b$, also $b>c$, $\therefore a>c$, hence $c-a$ is negative. (3.) . (4.) .

(5.) c or $\frac{a^2}{c}\cdot$ (6.) $-xyz(x+y+z)$.

(7.) Subtract 2nd from 1st and factor, etc.

(8.) $x^2=x-2$, $\therefore x^4=(x-2)^2$. Hence $x^5=x(x-2)^2=-3x^2+2x=-3(x-2)+2x=-x+6$.

(9.) . (10.) A \$648, B \$472, C \$416.

EXERCISE XXXIX.

(1.) 2 or $\frac{1}{7}$. (2.) See hint, question 10, Exercise xxiii.

Page 61. (3.) .

(4.) $(a+b)^2+(b+c)^2+2ac-2ac=(c+a)^2+2b(a+b+c)-2ac=$, etc. (5.) Multiply out and factor, etc.

(6.) . (7.) xy is greater.

(8.) $n^3+1 >< n^2+n$ or $n^2-n+1 >< n$ or $(n-1)^2 > 0$, etc.

(9.) .

(10.) $xyz = \frac{(b-c)(c-a)(a-b)}{abc}$, $(x+y+z) = \frac{(b-c)(a-c)(a-b)}{abc}$, $\therefore$, etc.

EXERCISE XL.

(1.) Apply $\frac{x^3-y^3}{x-y}$, etc. (2.) $\{ 2(a+b)^2-3(a-b)^2 \}^2$.

(3.) $(3x-4y+2z)(4x-5y+7z)$. (4.) 50 lbs., 30 lbs.

(5.) . (6.) $q=52$.

(7.) $1\text{st} = \frac{1}{(1+x)^2}$, $2\text{nd} = \frac{1}{(1-x)^2}$, $\therefore$ product $\frac{1}{(1-x^2)} \times \frac{1}{(1-x^2)} = (1+x^2+x^4$, etc.$)(1+x^2+x^4$, etc.$) = (1+x^2+x^4+$, etc.$)^2$.

(8.) . (9.) $(x+2y-2)^2$. (10.) $(a-c)(c-b)$.

MISCELLANEOUS EXERCISES.

A.

Page 62. (1.) $(4a-6)(4a-9)$.

(2.) Factor by difference of squares, $(x+1)(x-3)$.

(3.) $x=a+b$, $y=a-b$, expression $= \{ (x+y)^2+3xy \} \{ (x-y)^2-3xy \} = (4a^2+3a^2-3b^2)(4b^2-3a^2+3b^2)$, etc. (4.) $\frac{1}{16}(5p^2-q^2)(5q^2-p^2)$.

(5.) Apply $\frac{x^3-y^3}{x-y}$, $\therefore$ expression is divisible by $(x^3+x^2+4)-(x^3-2x+3) = x^2+2x+1$, etc.

(6.) Each side of $= n$ a cube. (7.) $3a+b+2c+d$.

(8.) $(x-m-n)(x-m+n)$. (9.) $(a-b)(b-c)(c-a)$.

(10.) Divide numerator of each by denominator, etc.

(11.) Dividend is divisible by $(x^2+\frac{1}{x^2}-2)$. Apply principle $\frac{x^3-y^3}{x-y}$; $x^4+2x^2+\frac{2}{x^2}+\frac{1}{x^4}+6$.

(12.) $(x+1)(x-1)(x-2)(x-4)$.

(13.) Add the equations, etc., $(a+b+c)^2$.

(14.) Apply difference of squares, $16(a-2b)(c+d)$.

(15.) $(x+y)(x^2-4xy+y^2)$. (16.) 42000.

(17.) $2x^5-4x^4$. (18.) .

Page 63. (19.) 1. (20.) $(x^2-y^2)(x^4+x^2y^2+y^4)$.
(21.) $\frac{1+3x}{1-x}\cdot$ (22.) $\frac{x^6+y^2}{x^2-y^2}\cdot$ (23.) $(x-1)(x-2)(x-3)$.
(24.) 0. (25.) .
(26.) Factors of 21 are ±1, ±3, ±7. Hence $(x+1)$
$(x-3)(x-7)$. (27.) Cube $x-\frac{1}{x}=1$, etc.
(28.) Tea $62\frac{1}{2}$ cts., sugar $6\frac{1}{4}$ cts. (29.) 512.
(30.) $y^2-2y^4+3y^6-4y^8$. (31.) $\frac{36a^4}{a^8-6561}\cdot$
(32.) Reduce fractions to mixed numbers and equate
remainders, etc., $x=10$.
(33.) $(x-2)(x+2)(x^2+4)(x+1)(x^2-x+1)$.
(34.) Apply difference of squares, $144x^2(1-4x^2)$.
(35.) $(2x-3)(2x-1)$.
(36.) Factor by difference of squares. (37.) .
(38.) $16ab^3$. (39.) .
Page 64. (40.) $(x+1)(x-3)(x+5)$. (41.) 25. (42.) $x-\frac{3y}{4}\cdot$
(43.) 884. (44.) $(x-19y)(x+17y)$. (45.) 45 cents.
(46.) $\frac{2a-3b+c}{2a-3c}\cdot$ (47.) $(x+9y+1)(x-4y)$. (48.) .
(49.) $6x^2+8xy+7y^2$. (50.) $x^2-\frac{8x}{15y}+\frac{1}{15y^2}\cdot$
(51.) . (52.) $(3x-4y-3z)(3x-4y+3z)$.
(53.) $(11x+13y)(9x-11y)$. (54.) $x=13$, $y=7$.
(55.) x^2+y^2. (56.) . (57.) 125.
(58.) 600. (59.) Add $=ns$, etc. (60.) $x=5$, $y=2$.
Page 65. (61.) $4x^2+4y^2+z^2-4xy-2xz-2yz$.
(62.) $(a+2b-3c)(a-b+2c)$. (63.) 130.
(64.) $(2xy+a+b)^2$. (65.) $4(x-y)(7x^2-2xy+7y^2)$.
(66.) $(x-1)(x+8)(x^2+7x+26)$. (67.) $x=1$.
(68.) a^2. (69.) $x=\frac{c(c-b)}{a(a-b)}$, $y=\frac{c(c-a)}{b(b-a)}\cdot$
(70.) $a^3+a^2b+ab^2+b^3$, $a^3+b^3+3a^2b+3ab^2+2a^2c+2b^2c+$
$4abc+4ac^2+4bc^2+8c^3$.
(71.) $a^3-125b^3+8c^3+30abc$. (72.) $(x^2z-1)(y^2z-1)$.
(73.) Apply $\frac{a^2-b^2}{a-b}$; result $3(b-a-6c+2d)$.
(74.) $x=3$, $y=2$.
(75.) Reduce to mixed numbers, etc., $x=8$.
(76.) $(x+y-1)(x-y-2)$. (77.) Apply $\frac{x^3-y^3}{x-y}$, etc.
(78.) . (79.) (80.) .

(81.) 24. (82.) $(3x+2y-4z)(2x-3y+5z)$.
Page 66. (83.) The second. (84.) $x-2$. (85.) 47. (86.) 16.
(87.) $x^2+3xy+2y^2$. (88.) $(a+b)(b+c)(c+a)$.
(89.) 7 and 2.
(90.) $(x^2-x-1)(x^2-7)$. (91.) $x=-1$, $y=1$, $z=1$.
(92.) $\frac{a^2}{b^2}$. (93.) $x=10$, $y=20$. (94.) $x^{2a+2b+2c}$.
(95.) Factors are $(b+c-a)(b+c+a)$, but $b+c+a=0$, $\therefore$ expression $=0$.
(96.) 14249. (97.) $x=a$, $y=2b$, $z=3c$. (98.) 70 miles.
(99.) $(x-y)^2(x^4+y^4+2x^3y+2xy^3+6x^2y^2)$.
(100.) $(2x-3y+1)(3x-2y+1)$.

MISCELLANEOUS EXERCISES.

B.

Page 67. (1.) $2abc(a+b+c)$. (2.) $x-3$. (3.) $x=5$.
(4.) $4a^2-5a-7$. (5.) 200 acres, 250 acres.
(6.) $(x+y+1)(x+y+2)$.
(7.) $(x^2+a^2+x-3)(x^2+a^2-x+3)$. (8.) 85.
(9.) $(1-x)(x^2-3x+4)$. (10.) $(a+b+1)^2(a^2+b^2+1)$.
(11.) $x^6-3x^4+2x^2-1$. (12.) H.C.F. $=3x-7$, $\therefore x=2\frac{1}{3}$.
(13.) $\frac{a}{a-1}\cdot$ (14.) $\frac{x^2-xy+y^2}{(x^2-y^2)(x+y)}\cdot$ (15.) 48.
(16.) $x=-6$. (17.) $(2a-1)(3a+2)(a-3)$.
(18.) 12 and 8. (19.) $6x^2+2x-5$.
Page 68. (20.) $(x-2)(x+2)(x^3+5)$. (21.) \$9.37½.
(22.) $\frac{6a^2b^2}{a^4-b^4}\cdot$ (23.) $x=2\frac{1}{2}$. (24.) $x=3$, $y=-2$, $z=5$.
(25.) $3x^3-2x^2+3x+2$. (26.) $\frac{1}{a}\cdot$
(27.) $(x-1)(x-1)(x-1)(x+1)(x^2-4x+1)$.
(28.) $\frac{m-2}{m+2}\cdot$ (29.) $a(a+5)(a^2-1)$.
(30.) $(a+b)(a+c)(b+c)$. (31.) 30, 40, 14.
(32.) x^2+x-2y^2+3y. (33.) $x=6$, $y=10$, $z=9$.
(34.) Reduce to mixed numbers and equate remaining fractions, $x=7$. (35.) $(5x+6)(4x-7)$. (36.) 17.
(37.) $x+6y+14z$. (38.) $x^2+12x-7$. (39.) $\frac{b}{8a}\cdot$
(40.) $(x+a+b)^3$. (41.) 180 yds., 205 yds.

Page 69. (42.) $\frac{8(y+2)}{8y-1}$. (43.) $x=2,\ y=3,\ z=4$.
(44.) The product of any four consecutive numbers increased by unity is a perfect square.
(45.) $\frac{18}{(x-1)(x+2)(x+5)}$. (46.) $x=5$. (47.) $x=4, y=15$.
(48.) 15 and 5. (49.) 7. (50.) . (51.) .
(52.) . (53.) . (54.) $(x+y-z)(x-3y+z)$.
(55.) $(4x+5y)^3$. (56.) $abcd(x-1)(x+1)(x+3)$.
(57.) $x=a+b+c$. (58.) 120 sheep.
(59.) $(2a+7b-3c)(2a-7b+3c)$. (60.) 0. (61.) b.

Page 70. (62.) $x=1,\ y=1$. (63.) .
(64.) $\{(a+2)x+a-1\}\{(a-1)x+a+1\}$.
(65.) $x=3a$. (66.) $A=48$. (67.) x^2-3x+2.
(68.) $(p+r)(q+s)$. (69.) $x^2+2xy+3y^2$.
(70.) . (71.) $(a-b)x$. (72.) .
(73.) $x=100,\ y=9$. (74.) 1.
(75.) $(1-5x+6x^2)(1-3x-4x^2)(1+3x-4x^2)$.
(76.) $x+1$. (77.) $\frac{6x^2+5x+1}{5x+1}$. (78.) 3456 and 2304.
(79.) . (80.) $(5x+12y)(8x-7y)$.
(81.) H. C. F. $=(a-3b+5c)$.
(82.) Apply difference of squares, $39x+y+2z$.
(83.) -102.

Page 71. (84.) $(9x+71)(5x-48)$. (85.) x^2+6.
(86.) $(3x+2)(64x^6-729)$. (87.) .
(88.) $2x^2+9y^2-5z^2+12yz-9xz-9xy$.
(89.) $(4x-y-7)(2x+5y+3)$. (90.) $a+bx+cx^2$.
(91.) 24. (92.) 226. (93.) -1.
(94.) $(4x+4y+2)(14x-5y)$.
(95.) $a-b$. (96.) . (97.) $x=6,\ y=1$.
(98.) $(a+b)^2-(a+b)(c+d)+(c+d)^2$.
(99.) $x=3,\ y=-1$. (100.) 100 acres.

MISCELLANEOUS EXERCISES.

C.

Page 72. (1.) $x+y+z$. (2.) $(4x+5y)(3x-7y)$. (3.) $x=3$.
(4.) $2a-c$. (5.) $(4x^2+4y^2+5z^2)^2$. (6.) 3005.
(7.) $x^{10}+x^9y+x^8y^2+x^7y^3+x^6y^4+x^5y^5+x^4y^6+x^3y^7+x^2y^8+xy^9+y^{10}$, remainder $2y^{11}$.

(8.) x^{a+b+c}. (9.) x^3+2x^2+3x+4. (10.) x^2+2x+3.
(11.) $x=1$. (12.) -166. (13.) $3x^2-5x+7$.
(14.) $1-x+x^2-x^3+x^4-x^5+x^6-x^7+x^8$. (15.) $2x+21$.
(16.) $(a-1)(a+1)(a-2)(a-4)$. (17.) x^2-y^2.
(18.) -63. (19.) $3(1+x)(1+y)(x-y)(1-xy)$.

Page 73. (20.) $\frac{3x^2+8x+4}{x^2+x+1}$. (21.) $\frac{4x}{x^2-1}\left(\frac{x^4-x^3-x^2+x+2}{x^6-1}\right)$.
(22.) $x=5$, $y=4$. (23.) $(x^2-1)^3(x+1)$.
(24.) . (25.) $3(x+1)(2x+3)$.
(26.) $\frac{3x^3+15x^2-71x-315}{(x+3)(x+5)(x+7)}$.
(27.) $(x-3)(2x+3)(x^2+3x+3)$.
(28.) x^2-x+1. (29.) \$1000. (30.) $(2y+7)(x+3)$.
(31.) $(x-1)^2(x^2+2x+3)$.
(32.) $(x+y-1)(x^2+y^2+x+y+1)$.
(33.) $x^8-x^7+x^5-x^4+x^3-x+1$. (34.) 3. (35.) 300.
(36.) . (37.) $1-3x+3x^2-x^3+x^4-3x^5+3x^6-x^7$.
(38.) $(x+8y)(y-x)$. (39.) 35.
(40.) $a^4+a^3p+a^2q+ar+s$. (41.) $(x^2+x+6)(x^2+x-2)$.
(42.) $\frac{x+4a}{x+2a}$. (43.) $(a^2-4ab+b^2)(a^2+4ab+b^2)$.

Page 74. (44.) -16. (45.) $3(x+y)+5(z+u)$.
(46.) Apply $\frac{x^3+y^3}{x+y}$. (47.) $x^4-4x^3+5x^2-4x+1$.
(48.) $x=-\frac{a+b+c}{3}$.
(49.) Reduce to mixed numbers, etc., $x=2$.
(50.) $-37a^3$. (51.) $a^8-2a^6+a^4+4a^3-1$. (52.) $\frac{x^2-b^2}{x^3-a^3}$.
(53.) $\frac{x-3}{x+3}$. (54.) .
(55.) Apply difference of squares.
(56.) $(x-1)(x^2+x+1)(2x-3)(2x+3)(4x^2+9)$.
(57.) Add $=ns$, etc., 0. (58.) . (59.) .
(60.) $x=3$, $y=-1$, $z=0$. (61.) \$2400.
(62.) 156 and 13. (63.) $y(y^2-3)$.

Page 75. (64.) 4 times. (65.) $\frac{x+1}{4}$.
(66.) $(2x+y)(x-2y)(2-x)(4+2x+x^2)$. (67.) $x=51$.
(68.) $x=4$. (69.) 1. (70.) $(x+p)(x^2-px+p^2)$.
(71.) $(y+2x)(y-2x)(2x+1)(2x-1)(4x^2-2x+1)(4x^2+2x+1)$. (72.) $80x^5(x^2-9)$. (73.) -6.

(74.) A \$25, B \$30, C \$35. (75.) $1+x+x^2$. (76.) .
(77.) $\frac{y^4-x^4}{xy^2}$. (78.) $4x^2+16x+11$. (79.) $x=1$.
(80.) $a=26$. (81.) $(a+b)(c+d)$. (82.) . (83.) $x=3$.

Page 76. (84.) 45. (85.) . (86.) $\frac{2}{b}$.
(87.) $n=10$. (88.) $d=5$. (89.) $(9x-47y)(12x-91y)$.
(90.) $4(1-x)(1+2x)(x+4)(3x+4)$. (91.) $3x+5$.
(92.) . (93.) $a+c$. (94.) 27. (95.) $4(a^2-b^2)^2$.
(96.) 1. (97.) $36x^6-217x^4+406x^2-225$. (98.) $1-x$.
(99.) $(12a+12b+c)(a-12b-12c)$. (100.) $x=2\frac{1}{4}$, $y=3\frac{1}{5}$.

MISCELLANEOUS EXERCISES.

D.

Page 77. (1.) . (2.) $a=c$ and $4b=c^2+8$. (3.) $x=6$.
(4.) $b^3=27c^2$. (5.) .
(6.) $a+b$ or $b+c$ or $c+a$ is equal to zero. (7.) $x=6$.
(8.) Factor in ordinary way. The product of 6 and 35 $=210$, and the difference of the factors of 210 will be co-efficient of 2nd term, or equal a; for example 1×210 is one pair, and $\therefore$ 2nd term would be 209 and expression $6x^2+209x-35$. The other co-efficients of x would be 103, 67, 37, 29, 23, 11, 1.
(9.) $c=-168$, $d=196$. (10.) $p=20$, $q=25$.
(11.) $x^2=a^2+2ad+d^2$, $y^2=$. etc.
(12.) Sq. root $=x^3+2x^2+5x-6$, etc. (13.) .
(14.) Write expression $(x^2-xy)^3-(x+2)^3$. Apply principle of difference of two cubes, $x^4-2x^3y+x^2y^2+x^3-x^2y+3x^2-2xy+4x+4$.
(15.) $y-\frac{1}{y}-2$. (16.) .

Page 78. (17.) $m^2-8m+11$.
(18.) $(3a+2b)(3a-2b)(x-3a)(x^2+3ax+9a^2)$.
(19.) . (20.) $x=\frac{1}{abc}$. (21.) $x=\frac{1}{a+b+c}$.
(22.) $=(\frac{1}{11})^2$. (23.) $\frac{x-y+xy}{y}$. (24.) $\frac{a^2+b^2+c^2}{abc}$.
(25.) . (26.) $x^2-\frac{x}{2}+\frac{4}{x^2}$. (27.) $\frac{16a^2x}{(x^2-a^2)(x^2-9a^2)}$.
(28.) 24. (29.) $\{(a+2)x+a-1\}\{(a+2)x+a-2\}$.

(30.) $R=31$. (31.) $(3ax+x+a)(ax-2x+a+2)$.

(32.) $(x^2+2xy-y^2)(x^2+xy+y^2)(x^2-xy+y^2)$.

(33.) Divide by Horner's method and the remainder will be the value, divisor $=2x^2-3x+4$, answer $=10$.

(34.) . (35.) . (36.) 4. (37.) $x=\frac{38}{41}$.

Page 79. (38.) $a=16$. (39.) $4x-12+\frac{9}{x}\cdot$

(40.) Reduce to mixed numbers and equate remainders in lowest terms, $x=2\frac{1}{2}$. (41.) $6x+3$.

(42.) $a=1$; $\frac{x^2+1}{(x+1)^2}\cdot$ (43.) $4x-5$. (44.) $m=6$ or $\frac{2}{3}$.

(45.) Reduce to mixed numbers, etc., $x=10$.

(46.) x^2-x+1.

(47.) See page 52, Ex. xxiii, question 10, etc.

(48.) $x^8+2x^6+3x^4+2x^2+1$. (49.) $x=-4$.

(50.) $(x+a)(x-b)(x-1)$. (51.) 35. (52.) 0, 0.

(53.) $4a^2b^2$. (54.) 1. (55.) $(x^m-y^n)(x^{2m}+x^my^n+y^{2n})$.

(56.) Divide by $(x-2)(x-5)$ and remainder is zero, $\therefore$ $a=74$, $b=120$.

Page 80. (57.) $a=0$, $b=-36$. (58.) .

(59.) Reduce 1st$=n$, $xz+xy=2yz$, divide by xyz, etc.

(60.) The former. (61.) $x=-\frac{1}{4}$.

(62.) $8x^2-4x-\frac{7}{4}$. (63.) 16.

(64.) $(a+b+c)^3=a^3+b^3+c^3+3(ab+bc+ca)(a+b+c)-3abc$, $\therefore$ $(a+b+c)^3=a^3+b^3+c^3-3abc$ since $ab+bc+ca=0$. (65.) .

(66.) Expression $=17x(x-y)^4$, $\therefore$ to make a complete cube we must multiply by $289x^2(x-y)^2$. (67.) .

(68.) Multiply by xyz, re-arrange terms and divide by abc, etc.

(69.) Write expression $\frac{b^2+c^2-a^2}{2bc}-1+\frac{c^2+a^2-b^2}{2ac}+1+\frac{a^2+b^2-c^2}{2ab}-1=0$, simplify, etc.

(70.) Reduce to form $a^3b^3+a^3c^3+b^3c^3=abc^4$, + etc., and divide by $a^3b^3c^3$, etc.

(71.) $\frac{z(x^2-y^2)}{xy}=a-b$, $\frac{x(y^2-z^2)}{yz}=b-c$; $\therefore$, etc.

(72.) See page 52, Ex. xxiii, question 10. (73.) 3.

Page 81. (74.) Factor expression, $\therefore$ factor required is $x-3$.

(75.) $x-7$. (76.) .

(77.) $(a^2+b^2)(c^2+d^2)=$, etc. (78.) .

(79.) $x^2y=z(x-z)^2+2yz(x-z)+zy^2$, $\therefore$ $z(x-z)^2=x^2y-2yz(x-z)-zy^2$ or $(x-z)^2=yz$.

(80.) Multiply by x and add unity to each side; $\therefore$ $(1\quad x)^2$, etc.

(81.) $b^2x^2-a^2y^2=ab^2x-a^2by$, etc.

(82.) $(a+b)x+ab=$, etc.

(83.) $x=-(y+z)$, $\therefore$ $-(y+z)\ (a^2-bc)+(b^2-ca)y+(c^2-ab)z=0$, $\therefore$ $(b^2-ca-a^2+bc)y=(a^2-bc-c^2+ab)z$, etc.

(84.) Subtract 2nd from 1st, divide by $y-z$, etc.

(85.) $1+a=1+\frac{x-y}{x+y}$ and $1-a=1-\frac{x-y}{x+y}$ $\therefore \frac{1+a}{1-a}=\frac{x}{y}$, etc.

(86.) Write $\left(\frac{1-x^2}{1-x}\right)\ \left(\frac{1-x^4}{1-x^2}\right)$, etc.

(87.) $x^2-xy+y^2=0$, $\therefore$ $(x-y)^2=-xy$, $\therefore$ expression $=x^2y^2(x-y)-xy(x-y)xy=0$, $\therefore$ x^2-xy+y^2 is a factor. (88.) .

(89.) Add the equalities, etc., but left hand will be $(x-y)\ (y-z)\ (z-x)$ which $=3abc$, etc.

(90.) $-z=\frac{1-x}{1-2x}$, which divided out gives $1+x+2x^2+$, etc.

(91.) $a^2-5a-14$. (92.) $\frac{1}{x^{m+1}y}$. (93.) .

Page 82. (94.) .

(95.) Let m be quantity subtracted, and instead of a, b, c, write $a-m, b-m, c-m$; $\therefore$ expression $=(a-m)^2-(b-m)\ (c-m)+(b-m)^2-(a-m)\ (c-m)+(c-m)^2-(a-m)\ (b-m)$. Simplify, etc.

(96.) . (97.) $x=1$.

(98.) Expression $=a(b^2+bc+c^2)+$, etc., $=(a+b+c)(bc+ca+ab)$, $\therefore$ $=0$. (99.) $p=25$, $q=-24$.

(100.) $-\frac{4}{3}$. (101.) $x=5$.

(102.) $A=2$, $B=3$, $C=1$, $D=1$.

(103.) Divide each by $x+a$, and remainders $=0$. Subtract, $\therefore$ $a(l-p)=m-q$, etc. (104.) $1-m^3-m^4$.

(105.) If reduced, $x+1$ or $x+2$ must be a factor, $\therefore x=-1$ or -2, and hence $p=3$ or $\frac{1}{2}$.

(106.) $(xy+xz+yz-x^2-y^2-z^2)$ is the other factor.

LIMITED